

# Geodesic Haptic Device for surface rendering

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## Abstract

*This paper describes a haptic device whose aim is to render the contact with a continuous and developable surface by means of the representation of a geodesic trajectory. Some preliminary tests conducted with industrial designers have showed that the trajectories performed while exploring the surface of a style product, for a qualitative evaluation, follows some particular trajectories that may be mathematically described as geodesic curves. In order to represent these particular curves a haptic strip based on a modular servo-controlled mechanism has been developed. Each module of mechanism allows us to control both the curvature and the torsion. This device, in respect to the commercial existing haptic devices, allows a hand-surface contact with the virtual model in real scale without artifacts, by self-deforming its shape in order to conform to the mathematical curve to render. The strip is 900 mm long and has 9 control points for bending and 8 control points for torsion. Due to these characteristics, it allows us to render exploration trajectories of several kinds of product shapes and dimensions. In order to allow users to fully explore an object surface, we have mounted the strip on a platform consisting of two MOOG-FCS HapticMaster devices, which permits 6DOF orientation of the strip and force feedback control. The paper describes the mechanism of the strip and the 6DOF platform starting from the empirical observations of the exploration of surfaces and highlights the problems encountered and the solutions adopted.*

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Virtual reality I.3.4 [Computer Graphics]: Graphics Utilities—Virtual device interfaces H.5.2 [Information Interfaces and Presentation]: User Interfaces—Haptic I/O

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## 1. Introduction

Although the first force-feedback devices were born for tele-operations in the 1950s [GT54], the first haptic devices used in a virtual reality environment have been developed at MIT in the 1990s [Pat90]. Since the appearance of the first haptic devices, most of the research and development activities have concentrated on point based force-feedback devices with limited workspace and force feedback hardly matching real industrial working conditions, and much less on surface-based tactile devices currently limited to proof of concepts of technological principles and very far away from prototypes allowing to demonstrate the possibility of addressing industrial applications. Anyway, the two aspects - force and tactile feedback- are kept separated mainly because of technical difficulties in considering both of them together. Most of force feedback technologies developed so far are focused on point-based contact, and in certain cases, especially in the medical field, have reached a high level of efficiency for

very vertical applications generally dedicated to the simulation of training activities. The evolution of both research and industrial applications seems to be based on multi-point contacts, which is a way to improve and enlarge the field of application grounding and extending on the current experimented technology. Some tentative devices providing the feeling of contact with a surface have been more recently developed [IYNK01]. They consist of small patches of sensors/actuators that can render very small surfaces and support limited input forces. In addition, although the mechanical architecture of these systems is simple, it is difficult to imagine that the resolution and the dimension of the contact surface may increase at an application level also in the medium term of 5 to 10 years, due to the difficulty of extending the prototype up to a resolution equivalent to a visual display of the same size.

The work presented in this paper concerns a haptic tool that aims to be a major advance in respect to point based

devices oriented to digital surface exploration, in the sense that it consists of a continuous physical strip that allows a continuous free hand contact along a line. The haptic tool has been developed within the context of a research project aiming at studying a system to render and modify shapes of industrial products (i.e., cars, domestic appliances) through the use of haptic interfaces. The final objective of the research is to develop and experiment novel interaction concepts and metaphors for product design that allow designers and stylists to handle digital shapes in more intuitive and easy manners, while preserving their natural manual skills and not limiting their creativity. The motivation relates to the fact that current digital tools for shape creation and evaluation, i.e. computer aided design (CAD) tools, are nowadays largely used for the design of products with aesthetic value. But these tools are too technical for designers; in fact, designers are more keen on expressing their ideas by examples using hands and crafting physical prototypes, rather than using the traditional mouse and keyboard interaction to describe and model shapes. While the first approach is very natural and intuitive for them, the second implies the specification and modification of drivers of the mathematical models generating the shape, which is complex and unnatural. Furthermore, the manipulation of surface control points is not an easy and intuitive way for developing new shapes during a creative process.

The first implementation of the haptic strip has shown some physical and technological limits. First of all the limited dimension of the domain of mathematical curves that we were able to represent. That version of the strip deforms and approximates only planar curves. The metaphor was to render the curve resulting from the intersection between an object and a plane, starting from a defined point that we identify as *anchor point*. By performing additional observations of the way designers interact with physical prototypes of new products during the evaluation of the characteristics of the shape, i.e., its quality, elegance, fluidity, we have noticed that they explore shapes along trajectories that are actually approximable with geodesic curves. Therefore, in order to extend the domain of curves that can be represented by the haptic planar strip, we have studied and developed an extension of the strip able to represent geodesic curves. We have redesigned the strip in a way such that it is now able not only to bend, but also to twist. This new version of the haptic strip allows the user to have a better feedback about the shape of the rendered object because the rendered curve approximates better the trajectories followed with the hand during the exploration of an object.

The analysis of styling activities performed by some designers has shown that new shapes are created and evaluated on the basis of a limited set of characteristic curves. On this basis we have developed a haptic interface that consists of an actuated strip that conforms to a selected curve on a shape. A modification of the strip causes a modification of the corresponding curve. The strip as interface for design has been considered by designers an effective tool for quick and in-

tuitive evaluation of the aesthetic quality of shapes. The paper is organized as follows: section 2 presents some related works on haptic interfaces, section 3 describes the haptic device for surface rendering consisting of an actuated strip that conforms to a selected curve on a shape, section 4 presents the extension of this strip adding torsion in order to render geodesic curves, section 5 illustrates the prototype of the geodesic strip, and finally section 6 draws some conclusions.

## 2. Related works

Haptic interfaces are devices that present tactile and force feedback to a human user who is interacting with a real or simulated object via a computer [JL06] in order to feel the virtual object's properties (i.e. texture, compliance or shape). There are mainly two types of haptic interfaces: tactile and force feedback display. Usually tactile-feedback devices have pin arrays or vibrators in order to stimulate the skin and commonly force feedback displays are powered by electric motors or other actuators in order to exert forces on user's hands. With regards to tactile feedback devices there are some interesting researches mainly based on vertical pins displacement for mid-scale virtual surfaces [IYNK01], or more recently the use of miniature pin array tactile module using elastic and electromagnetic force for mobile device [YKK\*09] which provides enough working frequency, output force and amplitude to stimulate the human's mechanoreceptors; the small and lightweight tactile display developed by [KKY\*09] is integrated into a haptic glove system the authors highlight the problem of embedding these types of tactile display modules into portable devices due to their intrinsic constraints. In [YRK09] a vibrotactile display is used for transmitting vector information in order to transmit directional information and spatial location (is not used for surface exploration). In [HIS09] it is presented a tactile display using airborne ultrasound: the prototype presented provides weak force for users to feel constant pressure, just sufficient for vibratory sensation. In [CPS\*09] the authors present a haptic interface capable of simulating forces experienced during abdominal palpation using pneumatic actuators. The only attempt to render a continuous contact along a line can be found in [BFCA09] and [BC08]. Regarding the force feedback display some of the most relevant haptic technologies [HAH\*04]: point-based devices like PHANToM (<http://www.sensable.com>), FCS-HapticMaster (<http://www.moog-fcs.com/robotics>), and the Haption-Virtuose (<http://www.haption.com>), multi-point based devices like the Haptex system (<http://haptex.miralab.unige.ch>), and the T'nD system [BC06].

## 3. Haptic device for surface rendering

The haptic device for surface rendering consists of an actuated strip that conforms to a selected curve on a shape. The

basic concept is to use as main user interface a force sensitive tangible strip, suspended in space in the position of a section of a simulated virtual object. The strip can actively shape and place itself in the appropriate position and orientation in the workspace. The psychophysical requirement to satisfy is to be able to explore and modify this part of a virtual object by means of touch. As a shape exploration tool, the system represents one of the very few attempts of a full, whole hand, encountered shape display, but with the limitation that it represents only a planar cut through the surface. Being a haptic device, the tangible strip is an output device in that the strip is an exploration device for the human hand and fingers to touch; and it is an input device, in the sense that the strip behaves as a physical item which can be shaped by hand like a physical bending spline. The haptic strip is implemented by means of a continuous physical spline that is actuated into the desired shapes by six equidistant relative actuators along its length.

The deformable strip is connected to a 6DOF platform consisting of two FCS-HapticMaster devices (<http://www.moog-fcs.com/robotics>) operating in a parallel configuration.

The positioning of the strip is controlled by defining the target position of the end point of each FCS-HapticMaster (HM) and the value of the pitch angle (controlled by digital servos). Figure 1 schematizes the haptic system.

An inverse kinematics algorithm has been implemented for positioning the strip. The algorithm takes as input the parameters of the plane the curve lies on (normal vector  $N$ ) and two points, named *anchor points*  $A_1$  and  $A_2$ , that have the characteristic of laying on the curve plane and having a position that does not depend on the curve shape. The direction  $A_1A_2$  is parallel to vectors  $D$  and  $D'$ .

These data are enough to compute the inverse kinematics of the strip in order to obtain the position of the end points of the HM devices and the pitch angle  $\alpha$  of the haptic strip. The curvature of the strip is controlled by six control points, each one actuated by a servo.

The computation of the shape of the haptic strip is based on an algorithm that takes as input the relative angles of each control point (Figure 2), from which, using a linear algorithm, is possible to compute the angle to impose to the arm of each servo controlling the haptic strip. In fact, the curvature of the strip on each control point depends on relative angle of previous, current and next control points. The resulting algorithm has a linear complexity  $O(n)$  where  $n$  is the number of servos used in the strip.

The new position of the cutting plane is completely specified. These data are sent to a geometric modeling module, which computes the new anchor point located on the surface of the object, and the new  $U$  and  $N$  vectors that at best fit to the object surface. The strip conforms its shape according to the new values: the new anchor point, the recomputed

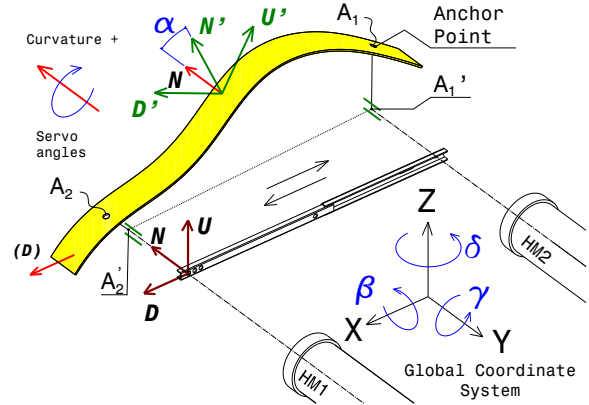


Figure 1: Representation of reference systems and parameters used for computing the inverse kinematics.

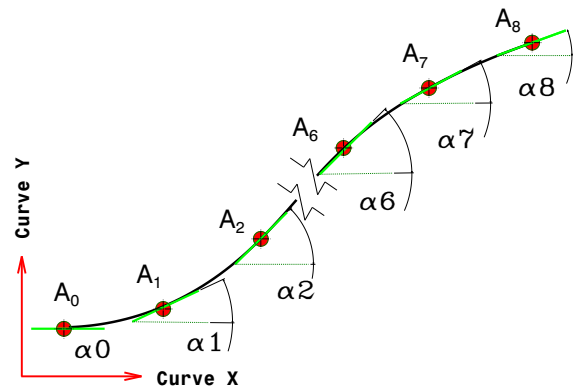


Figure 2: Representation of the six angles describing the shape of the planar curve.

vectors  $U$  and  $N$ , the position of the six control points of the haptic strip and the six angles describing the shape of the planar curve.

Concerning the haptic control and the force-feedback behavior of the device, the target position of each HM device is controlled using directional springs. For the HM device on the left, defined as *master*, three springs are set: one for each vector  $U$ ,  $N$  and  $D$ . For the HM device on the right, defined as *slave*, only two springs are set in directions  $U$  and  $N$ . Along direction  $D$ , the *slave* HM will follow the *master*, in order to avoid resonance problems between the devices. The springs with direction  $U$  are ten times stiffer than the others in order to give the feeling of being hardly snapped over the object surface.

### 3.1. Bending of the strip

The bending of the strip is controlled by a series of bending modules. Figure 3 shows schematically the geometry of a single bending module, described by the following parameters:

- $O$ : center of rotation of servo arm
- $C_0, C_1, C_2$ : control points of the part of the curve controlled by the module
- $SC_0, SC_2, SO, OC_1$ : constant-length segments
- $\alpha$ : angle  $\hat{S}OC_1$

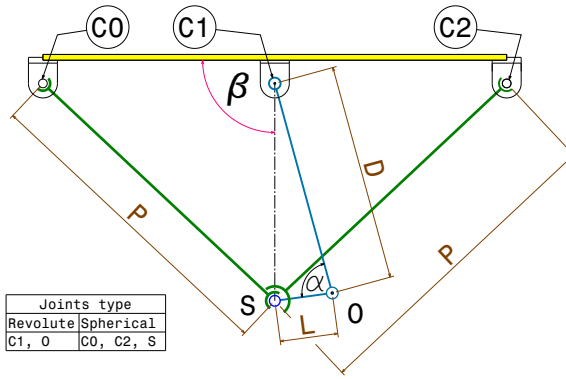


Figure 3: Bending module schema

Given  $\alpha$ , it is possible to compute the square distance:

$$SC_1^2 = L^2 + D^2 - 2LD \cos \alpha \quad (1)$$

from which we can compute:

$$\cos \beta = \frac{SC_1^2 + C_0C_1^2 - P^2}{2 \cdot SC_1 \cdot C_0C_1} \quad (2)$$

The magnitude of the curvature in  $C_1$  follows the relation:

$$K = \frac{\pi - 2\beta}{C_0C_1} \quad (3)$$

Values of  $\alpha$  are limited by the mechanical constraints of the device (Figure 4). The final result is:

$$\alpha = \arccos \left( -\frac{2W \left( W + \sqrt{W^2 - C^2 + P^2} \right) - C^2 + P^2 - L^2 - D^2}{2LD} \right) \quad (4)$$

Where  $W = C_0C_1 \sin(C_0C_1 \cdot K)$  and  $C = C_0C_1$ .

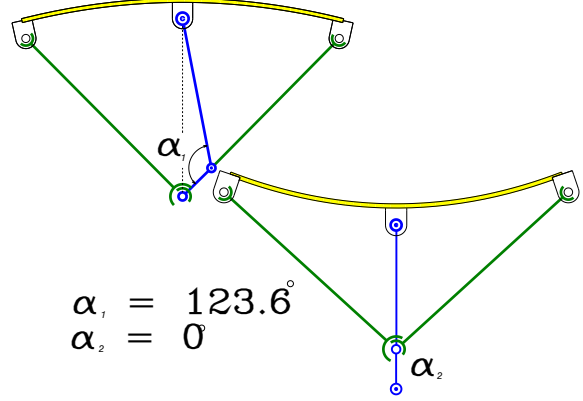


Figure 4: Values range for  $\alpha$

### 3.2. Prototype of the haptic strip

A first prototype of the haptic strip has been implemented in order to perform some tests and to get feedback from the target users of our system that, as mentioned before, are designers. The prototype consists of a plastic strip laying on four bending modules. The plastic strip is made of flexible PVC, its cross section is 50 mm x 2 mm with a total length of 900 mm.

Figure 5 shows the implementation of the bending module: it consists of the servo (1) connecting the lever and the control points (5) through two pushrods (2). Each bending module has one servo making the length of the link carrying it longer and shorter, and the two pushrods ending on its lever forming the V-shaped mechanism controlling the bending of that section of the plastic strip (3) symmetrically. The pushrods are linked through spherical joints to the lever and the shafts attached on each single control point. The bending servo makes exactly 180 degrees turn between making the distance from the lever end to the hanging pivot minimal and maximal, i.e. straight up the hanging link and straight down. The component (4), that carries on the servo for bending, acts as a pendulum.

The single module for bending has been designed taking into account some important considerations related to the use of sheet metal components: low inertia, light weight parts and low friction. The union of these components with the servo frame guarantees a very low rotation in the entire module. This solution provides a lighter and rigid module for bending.

Figure 6 shows the prototype of the final haptic strip implemented with nine actuators for bending. The haptic strip has been evaluated by some industrial designers participating to our research project in order to assess the conceptual design of the system. The designers appreciated the use of the continuous strip for evaluating the quality of the shape curvature. The only limitation concerned the impossibility

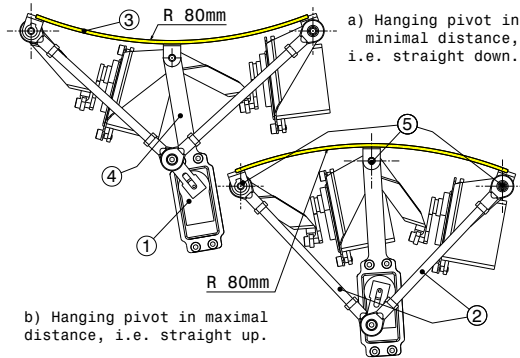


Figure 5: Bending module

of exploring those curves that are not planar. This stimulated the development of an extended version of the haptic strip with a geodesic version.

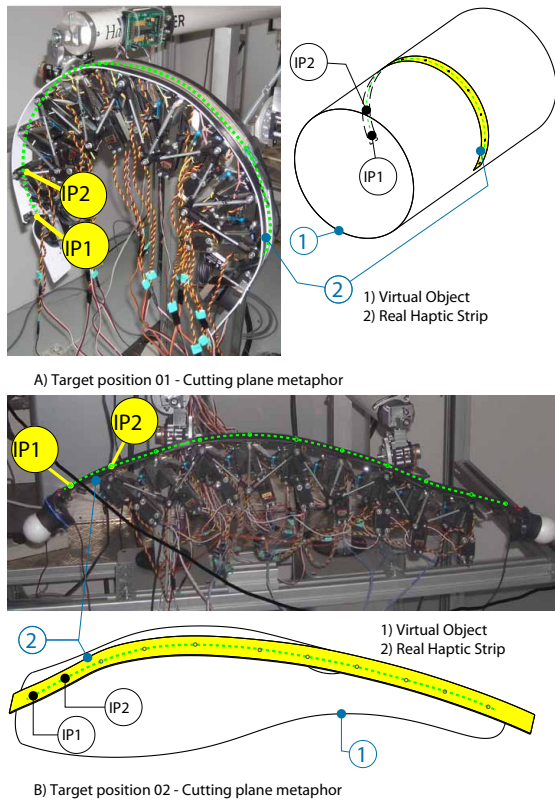


Figure 6: Prototype of the haptic strip

#### 4. Geodesic Haptic strip

In order to extend the domain of the curves that can be haptically rendered by the haptic strip we have implemented

a version of the strip able to render geodesic and longer curves. From a geometrical point of view, a geodesic curve is defined as the shortest path between two points in a curved space, for which the tangent vector field is parallel along this curve.

In order to show the concept using a strip to render a linear curve, a cylinder was chosen since it is known that geodesics on a cylinder are either straight lines (parallel to the axis of the cylinder) or circles (parallel to the base of the cylinder) or helices.

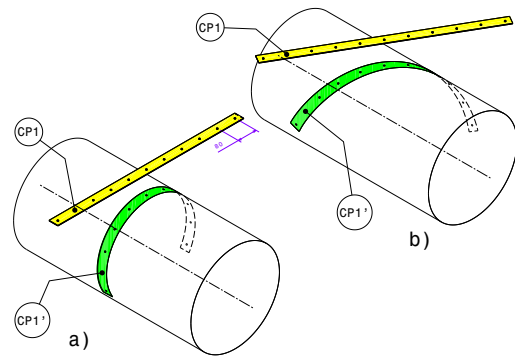


Figure 7: Examples of geodesic strips

Figure 7 shows both the plastic strip and the cylinder in which the strip is developed. Figure 7a shows a geodesic determined on the cylinder as a circle. Figure 7b shows the case of a general geodesic on a cylinder. The geodesic is indeed part of the helix connecting its beginning point to its end-point.

A geodesic is a particular curve on a surface which has a curvature that depends only on the surface. This is an important property of the geodesics that makes them attractive from a surface description or reconstruction point of view [ZZ94]. Geodesics can be used in a mechanism to check if a portion of a surface has constant Gaussian curvature. Additionally geodesics are useful in determining the geometrical properties of a surface (a fact supported by [Kre59]). This finds applications in geometric modeling and reverse engineering where a representation of a surface can be developed based on its geodesic.

The geodesic strip has been implemented by extending the first version including torsion. This new system allows the possibility to bend the strip using a planar system and to twist the strip using a spatial system.

#### 4.1. Kinematics for torsion

The kinematics of the geodesic strip is schematized in Figure 8, where the twist angles are highlighted. The servo angle  $\gamma$ , which is required to reach a given torsional angle  $\tau$  between two modules, can be computed as:

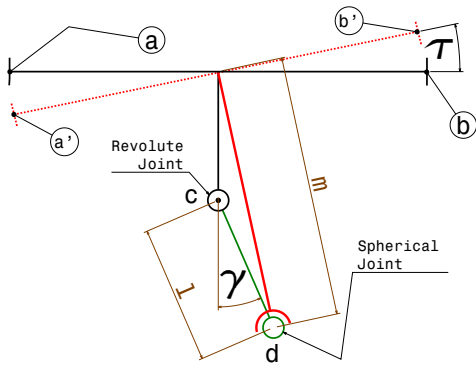


Figure 8: Kinematics schema

$$\gamma = \frac{m}{l} \sin \tau \quad (5)$$

Each module is independent from the others, and given the curvature values  $K_0, \dots, K_n$  (in equation (3)) and the tilt angles  $\tau_0, \dots, \tau_n$  the device is controllable with an algorithm of linear complexity.

Figure 9 shows the torsion module. A spatial module has been designed in order to provide the torsion required to render a geodesic curve. The torsion module is composed by a sheet metal bracket (2) carrying on the torsion servo (1) and a hinge (3) that is fixed through a revolute joint constrain to the control points (4). In between the servo lever and the hinge there is a spherical joint required to twist the next module. The mechanism can reach ten degrees ( $\tau$ ) while twist on each extremity of a single module. The resulting device is modular and easily upgradeable in case more degrees of freedom will be needed. In  $\tau_{02}$  ten degrees in clockwise torsion and in  $\tau_{01}$  ten degrees in counterclockwise torsion

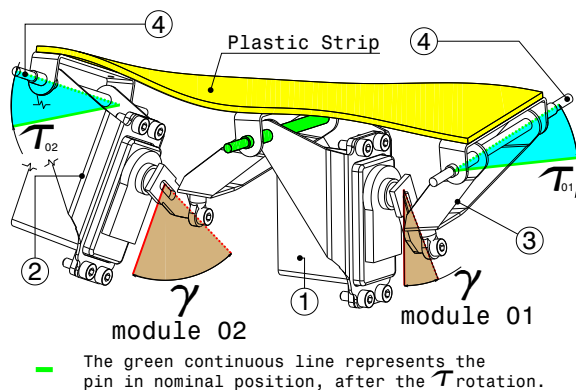


Figure 9: Torsion module

In order to prevent any undesirable rotation between the

servo drive and its sheet metal housing, an integrated module has been designed. The twist module includes the sheet metal bracket carrying on the torsion servo. Two bolts located in the side walls have been used to connect the servo frame with its sheet metal housing. Using an internal thread on each hole and using the two bolts, the servo drive is fixed by pressure. The union of these components guarantees a rigid module without compromising the rigidity of the module while twisting.

In order to render longer curves the geodesic strip consists of 900 mm long plastic strip controlled by 9 servo drives to bend and 8 servo drives to provide torsion. The strip mechanism works quite well in bending and in twist rotation. It is worth noticing that it is necessary the use of springs in order to balance the mechanism.

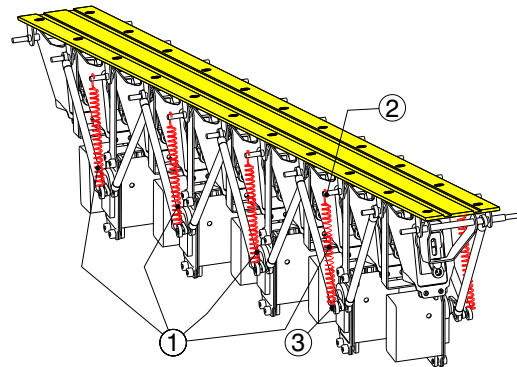


Figure 10: Tension springs

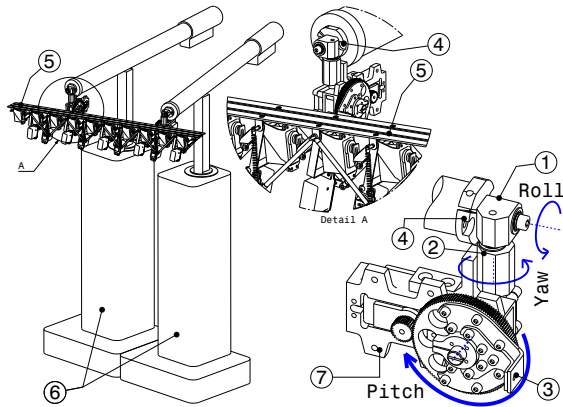
As can be seen in Figure 10, eight tension springs (four on the front (1) and four on the back, attached between the shaft (2) and the servo lever (3)) have been added in order to add a pre loaded tension.

The tension on each spring was calibrated in order to accommodate the plastic strip as horizontal as possible. The pre-load provided by the tension springs "helps" each servo when the strip is in nominal position (i.e. horizontal).

#### 4.2. Limits of geodesic mechanism as 6 DOF platform

Several possible configurations have been analyzed, using the reduction gear for tilting and the components needed to clamp the haptic strip mechanism on the two FCS-HapticMaster (HM) devices. The geodesic mechanism (5) is attached on the two HM devices (6) as shown in Figure 11, through the sheet metal component (3) mounted on the tilt mechanism (7). Three components have been required to connect each end effector of the HM device with the geodesic mechanism. The first one is the roll component (1), the second one is the yaw component (2) and the last is the tilt mechanism (7). The geodesic mechanism is clamped and unclamped using a quick clamping device (4). Using this

solution, 5 degrees of freedom in the positioning of the strip (3DOF translation, roll, and yaw) are provided by the HM devices and pitch is provided by two gear reduction systems.



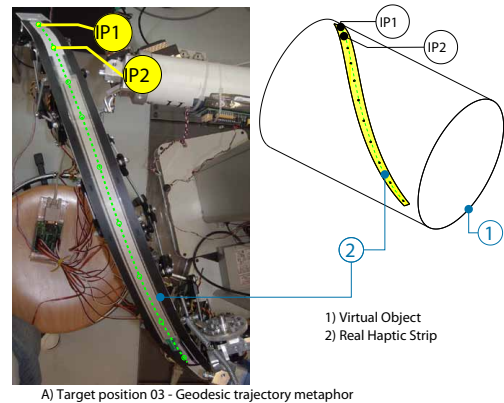
**Figure 11:** Links required to use the geodesic mechanism as a 6 DOF platform

This configuration provides five degrees of force feedback (during strip positioning, roll and yaw). Using two gear reduction systems to control the tilt permits direct and stiff connection between the HM devices. The servo drives have been selected so as to guarantee high reliability: the servo motor with titanium gears provides up to 2.35 Nm of continuous torque, and the gear reduction systems 5:1 are HS-5955TG manufactured by HITEC (<http://www.hitecrd.com/>). This allows us to get high stiffness and load capacity even when the user is applying pressure while exploring the geodesic strip. While tilting the geodesic mechanism is able to rotate from  $-55$  to  $+90$  degrees in the “X” axis. Regarding the yaw limits, the geodesic mechanism is able to reach  $\pm 62$  degrees in “Z” axis without any collision. With regards the roll limits, the geodesic mechanism rotates from  $+164$  to  $-164$  degrees around the “Y” axis.

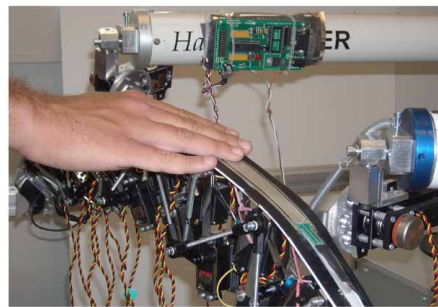
## 5. Prototype of the geodesic strip

On the basis of the design and simulations described in the previous sections, a prototype of the geodesic strip has been developed. Figure 12 a shows the geodesic prototype reaching the target position using the geodesic trajectory concept; obviously both the bending and the torsion modules are required to render the target curve while the strip is conformed. Figure 12 b shows the user while exploring the target curve using full hand.

The haptic strip has been integrated with a stereoscopic visualization system allowing users to see the virtual object and to interact with the haptic strip conforming to a selected curve of the virtual object in the same working space [BFCA09].



A) Target position 03 - Geodesic trajectory metaphor



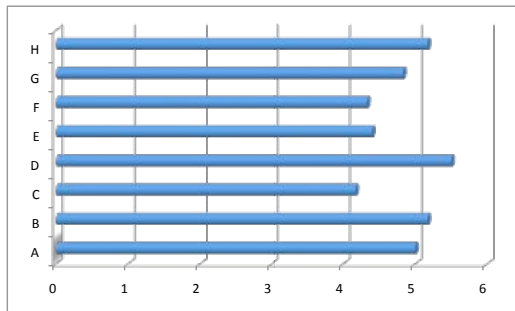
B) User while exploring the target curve using full hand

**Figure 12:** Prototype of the geodesic strip

## 6. Preliminary user testing of the geodesic strip

A preliminary test of the prototype of the geodesic haptic strip has been performed involving 6 participants, most of them working as professional designers in design companies involved in the research project. The participants were asked to perform the following task: position the geodesic strip in correspondence of the virtual curve to explore and to interact with the haptic strip in order to feel and evaluate the curvature. After the completion of the task, the participants were asked to score the following questions from 1 (bad) to 6 (good):

- Does the shape of the strip communicate thoroughly the idea of the shape of the object?
- How do you rate the effectiveness of the system in evaluating the shape quality?
- Was it easy to start using the system?
- Do you think the next time it will be easier using the system?
- How do you judge the interaction modality?
- How do you judge the overall comfort?
- How do you judge the visualization related to the haptic strip?
- Does the haptic perception of the surface reflect the visual one?



**Figure 13:** Results of preliminary test of the prototype

Figure 13 reports the results. We can conclude that the general impression about the system and its effectiveness (questions A, B) are very high. The participants thought that some training would improve the performances in using the system (questions C and D). The way of interacting with the overall system and the ergonomics aspects has been rated satisfactory (questions E and F). Some improvements should be considered. Finally, the co-location of the visualization and the haptic strip could be improved (questions G and H).

## 7. Conclusions

The paper has presented a novel haptic interface for rendering surfaces. The haptic interface consists of a continuous strip that deforms according to the curve belonging to a surface that it renders. The first version of the strip is 600 mm long and is able to render planar curves. The initial tests performed with users (industrial designers) have demonstrated that the use of the haptic strip for the evaluation of aesthetic shapes is effective and supports them in the appreciation of the aesthetic qualities of the shape. The strip is actually able to render a limited set of curves. The observation of the way that the designers use for touching and exploring physical objects surface has suggested the extension of the first haptic strip with the possibility of simulating also geodesic curves. The paper has described in details the implementation of the geodesic version of the haptic strip. The strip has also been extended in length in order to simulate the shape of larger surfaces. The strip allow users to feel an object curvature along a line by using their full hand. Despite the limitation of the available contact, the haptic strip is indeed a step forward in the development of haptic devices for the exploration of 3D surfaces.

## 8. ACKNOWLEDGMENT

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