

Visualizing Solar Dynamics Data – technical details

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Abstract—We discuss the technical details of the coronal magnetic field extrapolation on GPU using the Potential-Field Source-Surface (PFSS) model as described by Altschuler and Newkirk [2] and Schatten et al. [4].

1 CORONAL MAGNETIC FIELD EXTRAPOLATION ON GPU

While the magnetic field can readily be measured spectroscopically in the solar photosphere, this is unfortunately not feasible in the solar corona due to its much lower density. For this reason, extrapolation methods are needed to reconstruct the coronal magnetic field based on photospheric data. The simplest one—the Potential-Field Source-Surface (PFSS) model—was developed by Altschuler and Newkirk [2] and Schatten et al. [4]. It assumes that the coronal magnetic field is current-free,

$$\nabla \times \mathbf{B} = \mathbf{0}, \quad (1)$$

and thus can be represented by the gradient of a scalar potential,

$$\mathbf{B} = \nabla \psi. \quad (2)$$

Together with the divergence free condition of a magnetic field,

$$\nabla \cdot \mathbf{B} = 0, \quad (3)$$

the scalar potential has to fulfill the Laplacian equation

$$\nabla^2 \psi = 0. \quad (4)$$

For the solar corona, the Laplacian is transformed into standard spherical coordinates

$$x = r \sin \vartheta \cos \varphi, \quad y = r \sin \vartheta \sin \varphi, \quad z = r \cos \vartheta \quad (5)$$

with boundary conditions only at constant radii. At the photosphere, $r = R$, we have the Neumann boundary condition

$$\left. \frac{\partial \psi}{\partial r} \right|_{r=R} = M(\cos \vartheta, \varphi). \quad (6)$$

with $\vartheta = (0, \pi)$, $\varphi = [0, 2\pi)$, and the values from the corresponding magnetogram, $M(\cos \vartheta, \varphi)$. The outer boundary at $r = R_w = 2.5R$ is given by $\psi_r = 0$, where the field is assumed to be purely radial. The spherical magnetic field components then follow from

$$B_r = -\frac{\partial \psi}{\partial r}, \quad B_\vartheta = -\frac{1}{r} \frac{\partial \psi}{\partial \vartheta}, \quad B_\varphi = -\frac{1}{r \sin \vartheta} \frac{\partial \psi}{\partial \varphi}. \quad (7)$$

The relation between these spherical components and the corresponding Cartesian magnetic field vector

$$\mathbf{B} = B_r \mathbf{e}_r + B_\vartheta \mathbf{e}_\vartheta + B_\varphi \mathbf{e}_\varphi \quad (8)$$

is given by the spherical basis vectors

$$\mathbf{e}_r = (\sin \vartheta \cos \varphi, \sin \vartheta \sin \varphi, \cos \vartheta)^T, \quad (9a)$$

$$\mathbf{e}_\vartheta = (\cos \vartheta \cos \varphi, \cos \vartheta \sin \varphi, -\sin \vartheta)^T, \quad (9b)$$

$$\mathbf{e}_\varphi = (-\sin \varphi, \cos \varphi, 0)^T. \quad (9c)$$

In this work, however, we use spherical LCEA coordinates (r, μ, φ) where $\mu = \sin \theta = \cos \vartheta$ with latitude θ instead of colatitude ϑ as used in traditional spherical coordinates. The LCEA coordinates are related to Cartesian coordinates via

$$\Gamma : x = r\sqrt{1-\mu^2} \cos \varphi, \quad y = r\sqrt{1-\mu^2} \sin \varphi, \quad z = r\mu, \quad (10a)$$

$$\Gamma^{-1} : \varphi = \text{atan} \frac{y}{x}, \quad \mu = \frac{z}{\sqrt{x^2 + y^2 + z^2}}, \quad r = \sqrt{x^2 + y^2 + z^2}. \quad (10b)$$

The magnetic field vector \mathbf{B} , Eq. (8), is related to the gradient of the potential ψ with respect to LCEA coordinates in the following way:

$$\mathbf{B} = \begin{pmatrix} \sqrt{1-\mu^2} \cos \varphi & \frac{\mu\sqrt{1-\mu^2}}{r} \cos \varphi & -\frac{\sin \varphi}{r\sqrt{1-\mu^2}} \\ \sqrt{1-\mu^2} \sin \varphi & \frac{\mu\sqrt{1-\mu^2}}{r} \sin \varphi & -\frac{\cos \varphi}{r\sqrt{1-\mu^2}} \\ \mu & -\frac{1-\mu^2}{r} & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial \psi}{\partial r} \\ \frac{\partial \psi}{\partial \mu} \\ \frac{\partial \psi}{\partial \varphi} \end{pmatrix}, \quad (11)$$

In this work, we distinguish between *computational space* (c-space) spanned by the LCEA coordinates and *physical space* (p-space) corresponding to the Cartesian coordinates.

2 SPHERICAL HARMONICS

The solution to the Laplacian equation (4) in spherical coordinates is usually expanded into spherical harmonics. Following Altschuler and Newkirk [2], the magnetic potential ψ in the domain $R \leq r \leq R_w = 2.5R$ reads

$$\psi(r, \mu, \varphi) = R \sum_{n=1}^{\infty} \sum_{m=0}^n \{ \rho_n [g_n^m \cos(m\varphi) + h_n^m \sin(m\varphi)] P_n^m(\mu) \} \quad (12)$$

with

$$\rho_n = \left[\left(\frac{r}{R} \right)^n - a_n \left(\frac{R}{r} \right)^{n+1} \right] \frac{1}{a_n - 1}, \quad a_n = \left(\frac{R_w}{R} \right)^{2n+1}, \quad (13)$$

and Schmidt-normalized associate Legendre polynomials $P_n^m(\mu)$,

$$\int_{\mu=-1}^1 \int_{\varphi=0}^{2\pi} P_n^m(\mu) \begin{Bmatrix} \cos(m\varphi) \\ \sin(m\varphi) \end{Bmatrix} P_{n'}^{m'}(\mu) \begin{Bmatrix} \cos(m'\varphi) \\ \sin(m'\varphi) \end{Bmatrix} d\mu d\varphi = \frac{4\pi}{2n+1} \delta_{nm'} \delta_{mm'}. \quad (14)$$

Please note that we use μ instead of $\cos \vartheta$. The spherical harmonic coefficients g_n^m and h_n^m up to $n_{max} = 40$ are provided by GONG [1].

3 PFSS

As pointed out by Tóth et al. [5], the traditional spherical harmonics decomposition works reasonably well when the order of spherical harmonics is limited to be small relative to the resolution of the magnetogram. However, around sharp features, ringing artifacts become inevitable. Furthermore, spherical harmonics are global functions and

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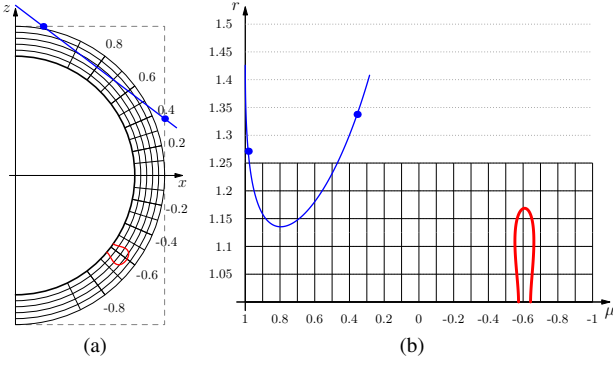


Fig. 1. Spherical curvilinear grid adjacent to photosphere (bold line). In p-space the longitude coordinate φ rotates around the z-axis (a). In c-space φ points into the drawing plane (b). Raycasting (blue) in p-space (a) would step along straight lines but along curved lines in c-space (b). We step along ray inside bounding box in p-space (dotted). Field line integration (red) in c-space allows for trivial and efficient point location. Respective vector field by gradient estimation in c-space.

their amplitudes depend on all of the magnetogram data. Since the basis of the spherical harmonics are the synoptic magnetograms, the global magnetic field is influenced by data that might have already changed significantly.

The finite difference method for PFSS has the advantage to be applicable also for local domains. In our application, after selecting a domain on the visible side of the Sun, we define a regular curvilinear grid bounded by either $\mu = \text{const}$, $\varphi = \text{const}$, or $r = \text{const}$. The lower boundary condition, $r = R = 1$, is defined by a line-of-sight HMI magnetogram which we resample for this grid. The other five faces follow from the spherical harmonics model using the coefficients by GONG.

The finite difference operator of the Laplacian equation $\nabla^2 \psi = 0$ in spherical LCEA coordinates (r, μ, φ) reads

$$\begin{aligned} \nabla^2 \psi_{i,j,k} = & \frac{\psi_{i+1,j,k} - \psi_{i-1,j,k}}{r_i \Delta r} + \frac{\psi_{i+1,j,k} - 2\psi_{i,j,k} + \psi_{i-1,j,k}}{\Delta r^2} \\ & - \frac{\mu_j}{r_i^2} \frac{\psi_{i,j+1,k} - \psi_{i,j-1,k}}{\Delta \mu} + \frac{1 - \mu_j^2}{r_i^2} \frac{\psi_{i,j+1,k} - 2\psi_{i,j,k} + \psi_{i,j-1,k}}{\Delta \mu^2} \\ & + \frac{1}{r_i^2 (1 - \mu_j^2)} \frac{\psi_{i,j,k+1} - 2\psi_{i,j,k} + \psi_{i,j,k-1}}{\Delta \varphi^2}. \end{aligned}$$

The boundary condition, Eq. (6), is realized by setting

$$\psi_{0,j,k} = \psi_{1,j,k} - \Delta r M_{j,k}. \quad (15)$$

Linearization of the scalar field

$$\psi_{i,j,k} \mapsto \Psi[(iN_{th} + j)N_{ph} + k] \quad (16)$$

yields a sparse linear system $A\Psi = b$ which we solve by the Krylov-type iterative method BiCGSTAB from the Cusp-library [3]. As initial guess, we set $\Psi = 0$ inside the computational domain.

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