

# 3D Shape Matching based on Geodesic Distance Distributions

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## Abstract

*In this work, we present a signature for 3D shapes which is based on the distribution of geodesic distances. Our shape descriptor is invariant with respect to rotation and scaling as well as articulations of the object. It consists of shape histograms which reflect the geodesic distance distribution of randomly chosen pairs of surface points as well as the distribution of geodesic eccentricity and centricity. We show, that a combination of these shape histograms provides good discriminative power to find similar objects in 3D databases even if they are differently articulated. In order to improve the efficiency of the feature extraction, we employ a fast voxelization method and compute the geodesic distances on a boundary voxel representation of the objects.*

Categories and Subject Descriptors (according to ACM CCS): I.5.3 [Computer Graphics]: Computational Geometry and Object Modeling —Geometric algorithms, languages, and systems

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## 1. Introduction and Related Work

The idea to describe 3D objects with distance distributions in order to perform shape matching is not new. Osada et al. [OFCD01], for instance, use the distribution of euclidean distances between two random points on the surface. The resulting descriptor is suitable to discriminate between different geometric shapes but is not invariant with respect to pose or articulation. Other approaches use geodesic distances to obtain such invariance. Examples include the work in [IAP\*08], where the distribution of the maximum geodesic distance from each vertex of a mesh to all other vertices (geodesic eccentricity) is used while [HSKK01] use the average of these distances (centricity). This distribution was also used in [GSCO07] combined with a local diameter function. In this work, we re-consider the idea in [OFCD01] and take the distance distribution of two randomly chosen points into account. However, we use geodesic instead of euclidean distances to provide pose-invariance and show that a combination of this distribution with the geodesic eccentricity and centricity distribution provides higher discriminative power than each of the individual descriptors alone. We refer to this combination as the GDD-descriptor, which stands for Geodesic Distance Distribution.

## 2. Computing Geodesic Distances

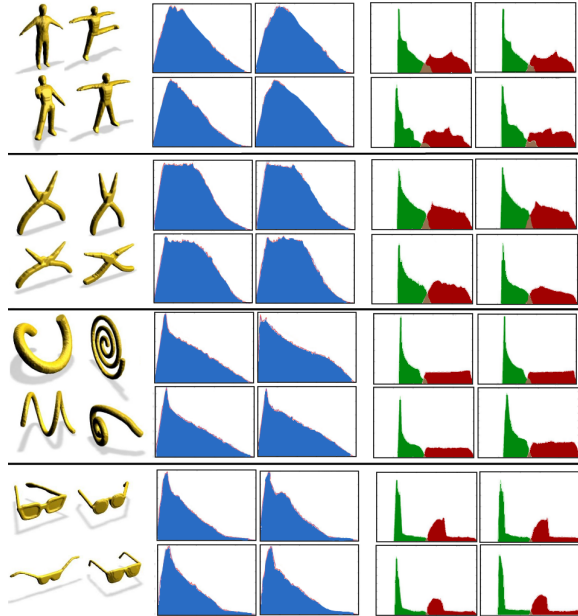
Our feature extraction requires a large amount of geodesic distance computations on 3D meshes. This operation is com-

putationally expensive and is often approximated by the shortest distance between vertices along the edges of the mesh, which makes the results highly dependent on the quality of the triangulation. To avoid this dependency, we accelerate the distance computation by performing a fast GPU-based boundary voxelization as a pre-processing step. A geodesic distance field around a voxel is obtained by Dijkstra's algorithm while the costs for propagating from one voxel to an adjacent one is depending on the type of neighborhood. A face-neighbor of a voxel has the cost of 1 while traveling to an edge- and a vertex-neighbor has a cost of  $\sqrt{2}$  and  $\sqrt{3}$  respectively. It is important that the voxel size is chosen sufficiently small so that two surfaces on the mesh, which are close together but do not touch each other, do not feature an adjacency in the voxel representation. This would spoil the approximation of the true geodesic distances. In our test database, a grid size of  $128^3$  has proven to be sufficient and the computation of a complete geodesic distance field around a voxel is in the magnitude of  $10^{-2}$  seconds.

## 3. Geodesic Distance Distributions

Let  $V$  be the set of voxels which represent an object and the geodesic distance between two voxels  $A$  and  $B$  is denoted as  $d(A, B)$ . We compute this distance for 100,000 pairs of randomly chosen voxels, which provides a histogram with the distribution of the geodesic distances. In order to normalize the range, we divide each measured distance by the maximum distance  $d_{max} = \max_{A, B \in V} d(A, B)$ . Examples of such

a distribution for different objects are shown in the blue histograms of Figure 1.



**Figure 1:** Resulting geodesic distance distributions for various objects. Pairs of randomly chosen points (blue), centrality (green) and eccentricity (red)

For the other two components of our feature vector, we need the following distances defined on a single voxel  $A$ :

$$d_e(A) = \max_{B \in V} d(A, B)$$

$$d_c(A) = \frac{1}{|V|} \sum_{B \in V} d(A, B)$$

The first one measures the geodesic eccentricity of a voxel, i.e the maximum distance to all other voxels while the second equation returns the average distance to all other voxels (centricity). In order to obtain a distribution of these metrics over the entire object, we compute them for 1,000 uniformly distributed voxels and normalize them by  $d_{max}$  in order to get a value between 0 and 1. Examples for resulting histograms for different objects can be seen in the green and red histograms of Figure 1. Each histogram consists of 100 bins in the range  $[0, 1]$  and we refer to them as  $D^{(p)}$ ,  $D^{(e)}$  and  $D^{(c)}$  for the randomly chosen pairs, the eccentricity and the centrality distribution respectively.

#### 4. Results in Similarity Searches

As we can see in Figure 1, the geodesic distributions are similar for same-class objects in different poses and distinguishable among different object classes. The examples also show a certain orthogonality between the different distributions.

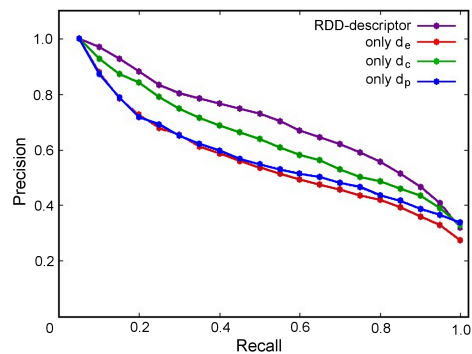
While  $D_c$  and  $D_e$  are quite similar for the human and pliers objects, the distribution of the randomly chosen points  $D_p$  differs among them. In case of the snake and the glasses, it is the other way round. This observation motivates the combination of the distributions to gain more discriminative power.

In order to compare two different shape descriptors, we first consider the individual differences of corresponding histograms and then obtain an overall difference value  $D_{GDD}$  by a convex combination of these differences:

$$D_{GDD} = \alpha |D_1^{(p)} - D_2^{(p)}| + \beta |D_1^{(e)} - D_2^{(e)}| + \gamma |D_1^{(c)} - D_2^{(c)}|$$

The difference between two corresponding histograms is obtained by the average absolute difference of corresponding bins. In experiments, we found that the optimal value for the weights is  $\alpha = 0.4$ ,  $\beta = 0.2$ , and  $\gamma = 0.4$ .

We tested the method on the database of the SHREC 2010, which contains 200 objects of 20 different classes whereas the objects within a class are differently articulated. The resulting precision-recall plots are shown in Figure 2.



**Figure 2:** Precision-Recall plot of each individual distribution and the combined method (GDD-descriptor).

The plot confirms that the RDD-descriptor renders significantly better results than using any of the three components alone. The average time for the computation of a descriptor, including the voxelization step, is around 5 seconds on an Intel Core i7 with an Nvidia GTX 285 graphics card.

#### References

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