EG UK Computer Graphics & Visual Computing (2018)
G. Tam and F. Vidal (Editors)

Voronoi Tree Maps with Circular Boundaries

(Extended Abstract)

Abdalla G. M. Ahmed†

Abstract

Voronoi tree maps are an important milestone in information visualization, representing a substantial advancement of the original tree maps concept. We address a less-studied variant of Voronoi tree maps that uses multiplicative-weighted Voronoi diagrams. We highlight the merits of this variant, and discuss the difficulties that might have discouraged further exploration, proposing insights for overcoming these difficulties.

1. Introduction

Introduced by Shneiderman and Johnson [JS91], *tree maps* represents an invaluable invention in information visualization of hierarchical data. The basic idea is to recursively partition the display area into regions whose areas are proportional to given weights of the data. The key advantage of tree maps is that they utilize the full two-dimensional space for displaying information.

A lot of research followed the seminal 1991 paper, and many variants were proposed to improve the model. A significant advancement, by Balzer et al. [BDL05], was the introduction of Voronoi tree maps that solved important issues of the rectangular models. The idea is to construct a Voronoi-like tessellation with proportional areas to the given weights. A Voronoi tessellation is a partitioning of space that assigns each point in the space to the nearest site from a given set of sites. For tree-mapping, a weighting mechanism is needed to control the area of the Voronoi-like cells. Balzer and Deussen [BD05] discussed two alternatives: additive weighting (AW), by adding a fixed weight to the Euclidean distances, and power weighting (PW), which adds the weights to the squared distance. AW produces a very appealing organic look, with parabolic boundaries between cells, but is considerably more difficult to control and represent. PW, with its linear cell-boundaries, trades the aesthetic look for efficiency, and very efficient computation algorithms were developed subsequently [NB12].

A different weighting mechanism for Voronoi diagrams, multiplicative weighting (MW), remained almost unexplored for tree mapping. In MW Voronoi diagrams, distances from sites are divided by the weight of the site. This leads to circular-arc boundaries between the cells, which seems quite attractive for tree-mapping, combining the organic look with the ease of representation. On the downside, however, is the fact that multiplicative weighting does not necessarily produce contiguous regions of the cells as does the additive (linear or power) weighting: there is a chance of having some fragmented cells, or some cells being islands inside other cells. The work of Reitsma et al. is the only tree-mapping one we are familiar with that considered MW, and they did not offer a solution to the mentioned problems. Another difficulty with multiplicative Voronoi tessellation is the high (quadratic) time complexity of known computation algorithms, compared to regular Voronoi diagrams or power diagrams [AE84].

2. Balzer's Algorithm

At a later time, an interesting algorithm for computing weighted Voronoi-like diagrams was presented by Balzer and colleagues [BH08, BSD09]. In a nutshell, the domain is discretized into a set of points that are assigned randomly to the sites, and then the points are exchanged between pairs of sites in accordance to the (weighted) distance, until no more exchanges take place. Thus, the algorithm starts with an invalid Voronoi diagram, and ends up with a valid one. The algorithm is typically applied alternately with an optimization algorithm such as Lloyd's, in which each site is moved to the centroid of its assigned region.

While the original Balzer algorithm is considerably slow, with quadratic time complexity, it is truly versatile, being able to handle a wide range of weighting mechanisms. Quite notably, even though the algorithm came from the same group of researchers who introduced Voronoi tree maps, we are unaware of any use of it for generating tree maps, possibly due to the existence of faster alternatives.

Most recently, specifically last year in this conference, Ahmed and Deussen proposed an accelerated implementation of Balzer algorithm that offered very competent performance [AD17]. The idea is to consider only the (unweighted) Voronoi neighbors, instead of all the pairs of sites. Even though the idea is slightly flawed, as we will discuss below, this improvement brought the original powerful algorithm into focus, and was the primary source of our inspiration.

† Email:abdalla_gafar@hotmail.com

DOI: 10.2312/cgvc.20181214

3. Balzer Algorithm with MW

Our primary observation is that, combined with Lloyd optimization, Balzer algorithm is capable of eliminating the aforementioned problems (islands and fragmentation) with MW Voronoi tessellation. The key insight to this is found in [AE84, Observation 2–1]:

> "Let $S = p,q$ consist of two weighted points in the plane and let $w(p) < w(q)$. Then the region of influence of p is the closed disc with center $\left(\left(w^2(p)\,p - w^2(q)\,q\right) / \left(w^2(p)\,p - w^2(q)\,q\right)\right)\ldots$"

Upon careful inspection of this formulation we see that the center is always away from the other site, hence Lloyd optimization would move the sites further apart, pushing any islands of smaller sites towards the cell edges of larger sites. This should effectively eliminate islands and fragmentation.

4. Ahmed's Implementation with MW

With Ahmed's accelerated implementation framework of Balzer algorithm [AD17], multiplicative Voronoi diagrams would represent an attractive solution for tree maps. The computation of the sorting keys for the exchange process turns to be surprisingly simple; the sorting key of a point is simply the ratio of its distances to the two sites. This follows directly from the definition of MW. Thus, the expected performance is similar to that in [AD17] for power cells, except that there is a division operation involved.

However, Ahmed's model is slightly flawed, since the topology is different for weighted and unweighted Voronoi tessellation. This difference is not that significant for the stippling application showcased in their paper, hence, apparently, the flaw went unnoticed. In contrast, wrong identification of neighbors matters for tree maps, since the focus is on the regions, not the sites. Fortunately, we were able to find a working idea to replace the auxiliary Delaunay triangulation, relying on the topology of the underlying point distribution. Two sites are neighbors if some of their assigned points are adjacent. Thus, a sweep over the pixels would be sufficient to retrieve neighbor information. Once found, the rest continues as in [AD17].

5. Conclusions and Outlooks

We coded a basic implementation of the adapted Ahmed's implementation of Balzer's algorithm to test the preceding insights that the modified algorithm effectively and efficiently removes islands and fragmentation. The initial results are promising; see Figure 1. The resulting tessellation arguably looks similar to AW Voronoi tree maps, but representing and reproducing circular boundaries is supposedly easier and simpler than parabolic boundaries. There is a lot of details to investigate further, making this concept a suitable research project for a master-level student, we think. We briefly outline some of the aspects that need to be investigated:

- A formal proof and an empirical evaluation are required to validate the approach.
- The required resolution for identifying neighbor relationships is not necessarily the same as that for the exchange process. "Minkowski's Theorem" is the keyword for a research on the optimal resolution.

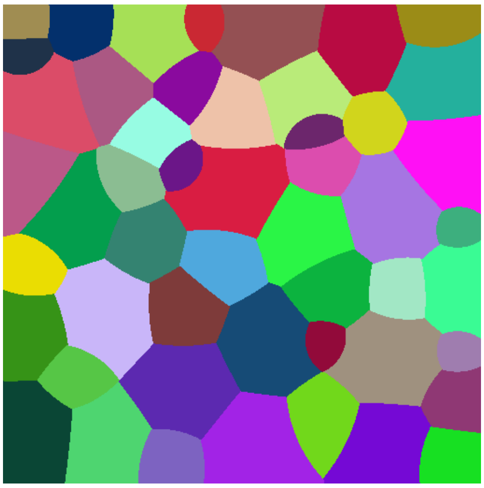

Figure 1: *A multiplicative-weighted Voronoi tessellation of 50 sites with random weights between 1 and 5, free of islands and fragmentation artifacts. This 512×512 plot was computed in 2.2 seconds in a 2.5 GHz CORE i5 laptop.*

- We only tested a non-hierarchical tessellation. Again, deciding the optimal resolution is needed across levels of the hierarchy.
- The edges are circular, hence they should be easy to represent. However, extracting these edges requires explicit assignment of weights to the sites. This seems tricky, as far as we can see.
- Finally, a thorough comparison with AW and PW is required.

References

[AD17] AHMED A. G. M., DEUSSEN O.: Capacity Constrained Voronoi Tessellation Revisited. In *Computer Graphics and Visual Computing (CGVC)* (2017), Wan T. R., Vidal F., (Eds.), The Eurographics Association. 1, 2

[AE84] AURENHAMMER F., EDELSBRUNNER H.: An optimal algorithm for constructing the weighted voronoi diagram in the plane. *Pattern Recognition 17*, 2 (1984), 251 – 257. 1, 2

[BD05] BALZER M., DEUSSEN O.: Voronoi treemaps. In *Proceedings of the Proceedings of the 2005 IEEE Symposium on Information Visualization* (Washington, DC, USA, 2005), INFOVIS '05, IEEE Computer Society, pp. 7–. 1

[BDL05] BALZER M., DEUSSEN O., LEWERENTZ C.: Voronoi treemaps for the visualization of software metrics. In *Proceedings of the 2005 ACM Symposium on Software Visualization* (New York, NY, USA, 2005), SoftVis '05, ACM, pp. 165–172. 1

[BH08] BALZER M., HECK D.: Capacity-constrained voronoi diagrams in finite spaces. In *Voronoi Diagrams in Science and Engineering* (2008). 1

[BSD09] BALZER M., SCHLÖMER T., DEUSSEN O.: Capacity-constrained point distributions: A variant of lloyd's method. In *ACM SIGGRAPH 2009 Papers* (New York, NY, USA, 2009), SIGGRAPH '09, ACM, pp. 86:1–86:8. 1

[JS91] JOHNSON B., SHNEIDERMAN B.: Tree-maps: A space-filling approach to the visualization of hierarchical information structures. In *Proceedings of the 2Nd Conference on Visualization '91* (Los Alamitos, CA, USA, 1991), VIS '91, IEEE Computer Society Press, pp. 284–291. 1

[NB12] NOCAJ A., BRANDES U.: Computing voronoi treemaps: Faster, simpler, and resolution-independent. *Comput. Graph. Forum 31*, 3pt1 (June 2012), 855–864. 1