

PROBLEM

Simulating fluids using SPH has been an active research topic in computer graphics for a decade. However, prior research on this topic has mainly focused on large-scale liquid simulations [1]. Despite the advent of SPH on different fronts of liquid simulation, simulating fine-scale, high-detail motions such as those involving droplets interacting with a solid surface are still hard to capture, and remains largely unexplored (Fig 1). Such microscopic-scale simulations can be used in a wide range of applications, such as special effects in both film and television, driving simulations, 3D printing, as well as physics, engineering, and medical simulations (e.g., respiratory droplet simulations) [2].

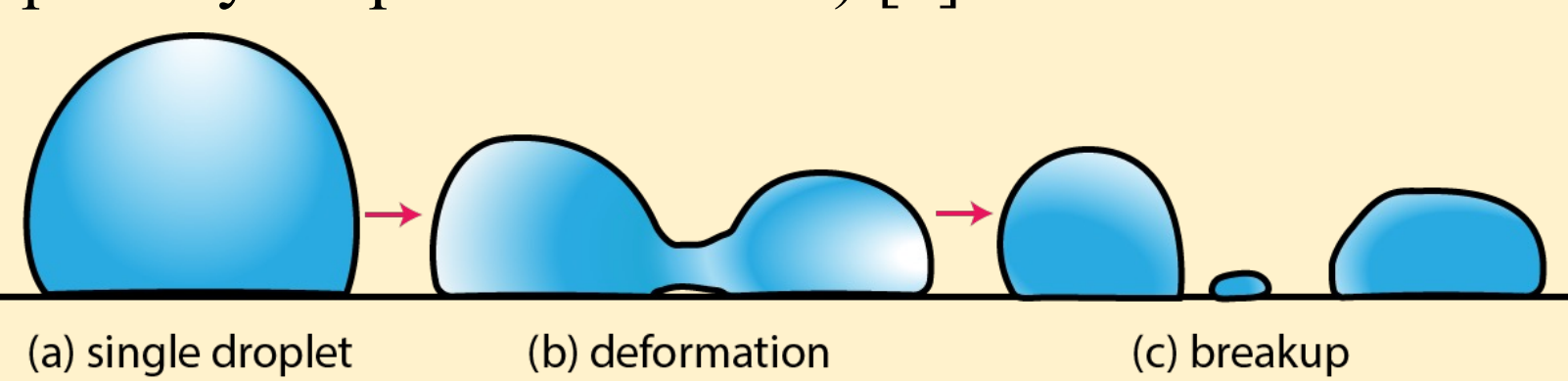


Fig 1: Droplet dynamics when impacts upon a flat solid surface, which include deformation and break up.

RELATED WORK

In our method, we employ the improved free surface ISPH model presented in [3], which presented an analytical solution that applies the pressure Poisson equation (PPE) to impose a Dirichlet boundary condition for pressure at the free surface. Then, the surface tension forces are formulated based on the equations presented in [4]. The surface tension forces are used to couple the cohesion force and minimise the area of the free surface within the ISPH framework. On the triple line surface tension (Fig 2), we computed the contact angle and contact force from the surface energy balance of Young's equation between the free surface and droplet-solid interface as presented in [5].

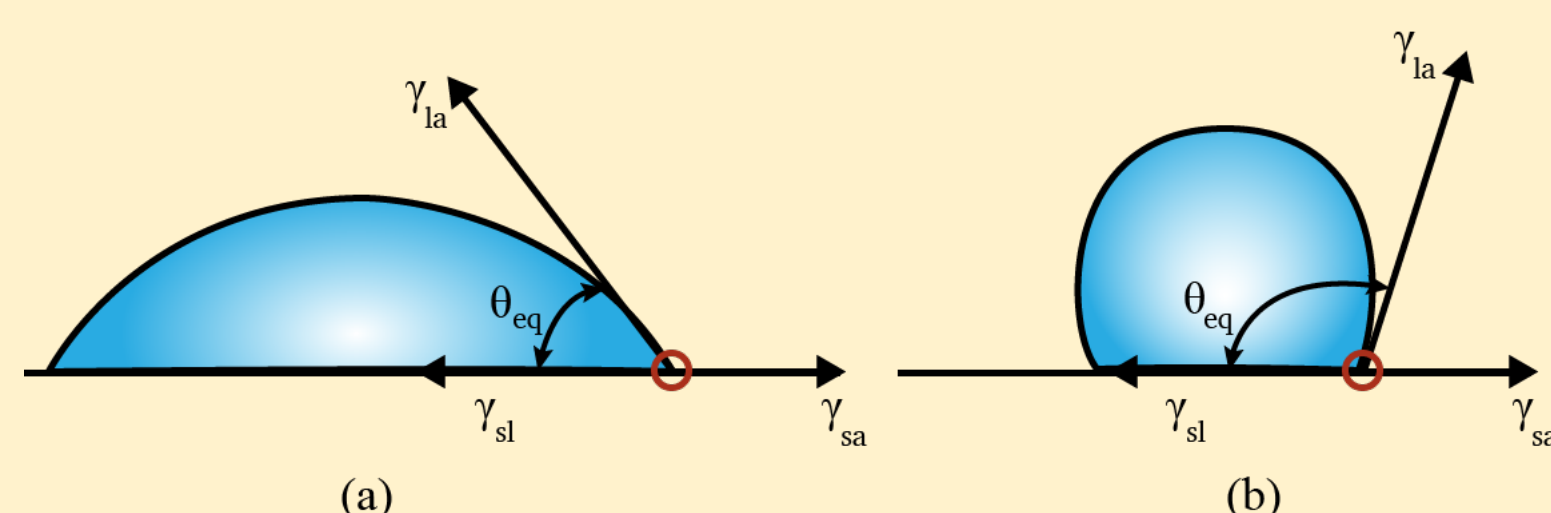


Fig 2: A droplet on a flat solid surface. The arrows indicate the interface tensions coefficients at the liquid-air (la), the solid-liquid (sl), and the solid-air (sa) interface.

OVERVIEW

In our method, we employ the improved free surface ISPH model presented in [3] and the surface tension forces formulated in [4]. The surface tension forces are used to couple the cohesion force and minimise the area of the free surface within the ISPH framework. This would model the inter-particle force as an attraction-repulsion function, which is responsible for causing short-range repulsion and long-range attraction. On the triple line surface tension, we computed the contact angle from the surface energy balance of Young's equation between the free surface and droplet-solid interface. Then, a contact force is calculated between the droplet and solid in a way similar in fashion to the one by [5], and plugged into our ISPH model. We demonstrate the efficacy of our model by simulating a droplet impacts upon a solid surface where spread, stretch and break up droplet deformations are realistically captured.

METHODOLOGY

Our model creates the coalescence, spreading, and break up deformations that occur when a liquid droplet when collides with a flat solid surface. We model the attraction-repulsion forces within an improved free surface ISPH framework presented [3], where the surface tension forces formulated in [4]. The macroscopic phenomena of surface tension appear from attractive forces between particles that experience an unbalanced molecular cohesive force in the boundary regions (which can either be at a liquid-air, liquid-solid, or solid-air interface). Particles in this region resist deformation, and actively minimise the area of a free surface. Therefore, surface tension can be represented in terms of the pressure difference across the surface, according to Laplace's Law,

$$\Delta P = \gamma \cdot \kappa$$

Where ΔP is the pressure difference, γ is the surface tension coefficient, and κ is the surface mean curvature. κ is related to a unit normal of the surface as $-\nabla \cdot \mathbf{n}$. The surface tension force can be formulated as:

$$F_i^{st} = \gamma_i \kappa_i \mathbf{n}_i$$

Hence, the normal can be calculated by applying the SPH approximation to the gradient of the smoothed colour field as:

$$\mathbf{n}_i = \sum_j \frac{m_j}{\rho_j} \nabla W(\|p_i - p_j\|, h)$$

Where $\nabla W(\cdot)$ is the smoothing kernel and h is the smoothing radius. m_j denotes mass and p_j denotes the position of particle j and ρ_j is the density.

Therefore, the surface tension force can be expressed as follows based on the formulation presented in [4].

On the triple line surface tension, the contact angle is calculated from the surface energy balance of Young's equation as:

$$\gamma_{sa} - (\gamma_{la} \cos \theta_{eq} + \gamma_{ls}) = 0$$

where θ_{eq} ($0 < \theta_{eq} < \pi$) is the stable contact angle, and γ_{ls} , γ_{sa} and $\gamma = \gamma_{la}$ are tension coefficients for the liquid-solid, solid-air, and liquid-air surfaces, respectively. When θ_{eq} is small (close to zero), the solid surface is called hydrophilic, and the liquid surface tends to spread flat. The solid surface is called hydrophobic if θ_{eq} is large, and the liquid tends to bead up on the surface (see Fig 2). To formulate a contact force upon impact, Young's equation is set to be equivalent to the contact angle such that the contact force acting on the triple line region is zero.

$$F_c = \gamma (\cos \theta_{eq} - \cos \theta_{dyn})$$

where θ_{dyn} is an arbitrary unbalanced angle, which is balanced when the equilibrium value, $\theta_{dyn} = \theta_{eq}$ is achieved. The contact angle θ_{dyn} can be computed in the simulation using the surface tangent and normal vectors. For more details on this, you can see [5]. The force F_c is applied to the ISPH particles in the triple line region, where we apply no-slip conditions on the solid surface.

RESULTS

When a droplet impacts a solid surface, it can undergo different types of deformation. Our method captures a range of such droplet deformations, in particular spreading, break up, and coalescence. These deformations can be observed in Figs (3, 4). Fig 3 shows a simulation of a free falling 2cm droplet of 3K SPH particles using our method, where the sequence illustrates the droplet dynamics at various stages during the impact, where the droplet bounces off before spreading on the surface. Fig 4 illustrates three types of deformations; namely spreading, coalescence, and break up, which result from the impact of a liquid droplet into a flat solid.

Moreover, depending on the droplet's parameters and surface conditions, when the droplet impacts upon a solid surface, deformations such as rebounding and splashing happen. The emission of tiny droplets in the impact direction is called impact splashing. In our current work, impact splashing is not captured by the simulation, and as a future direction, we will explore further the possibility to simulate and reproduce phenomena such as impact splashing for varying values of surface roughness.

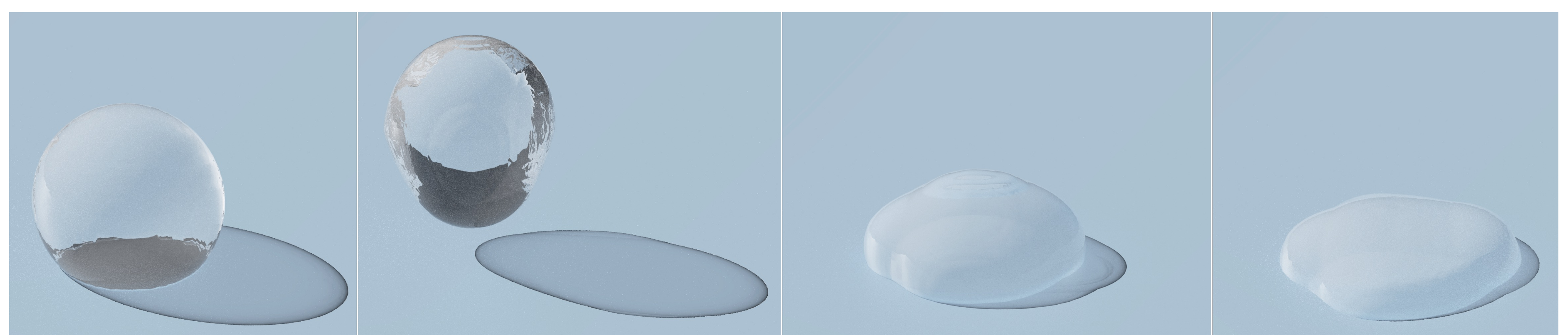


Fig 3: Our simulation of a droplet impacts upon a flat solid surface. The sequences from left to right correspond to times: $t=$ 30ms, 40ms, 90ms, and 100ms

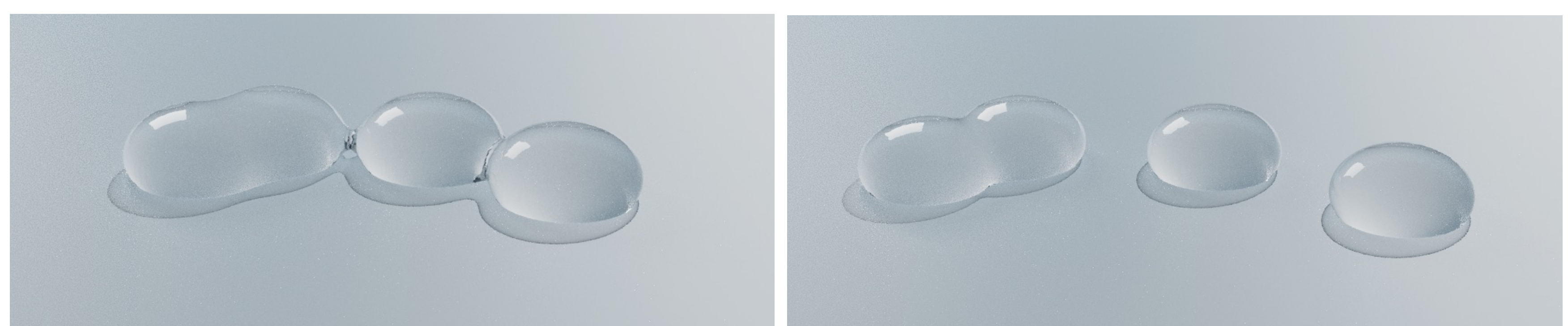


Fig 4: Deformations as a result of the impact of a liquid droplet into a flat solid. From left to right, spreading, coalescence, and break up can be captured using our model.

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