Interactive Simulation for Multimodal Virtual Environments

Dinesh K. Pai
Dept. of Computer Science
U. British Columbia
Vancouver, Canada

Objectives

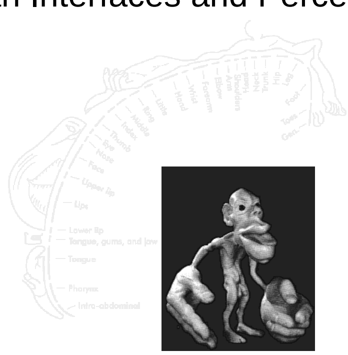
- Learn how to produce multimodal cues for interaction in VR
- Describe physically-based techniques for visual,auditory,haptic simulation
- Recommend specific techniques, rather than exhaustive survey (personal bias?)
- Understand mathematical foundations of methods, at an intuitive level

Outline

- Introduction
- Geometric techniques
- Rigid body dynamics and contact
- Contact sound simulation
- Deformation simulation

Introduction

Human Interfaces and Perception



Human Perception

- Most real world phenomena are multimodal: produce correlated visual, auditory and haptic cues
- Human perception has evolved to take advantage of these correlations
  - robustness in presence of noise, occlusion, etc.
  - multitasking
- Multimodal stimuli important for presence

Example: The McGurk effect

- [http://kahuna.psych.uiuc.edu//ipl/]

Some (Virtual) Realities

- Manipulating a “rigid” object
- NEEDS: visual motion; dynamics; impact, sliding and rolling forces; sounds; temperature,...

Some (Virtual) Realities

- Human avatars, articulated objects
- NEEDS: Multibody dynamics, deformation, sounds of footsteps, ...

Some (Virtual) Realities

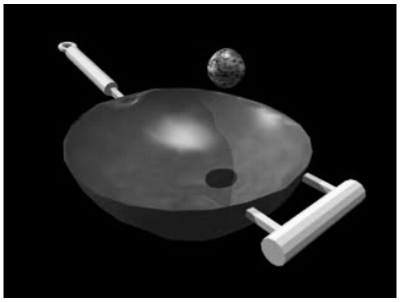
- Soft objects (e.g., furniture, cloth, other humans)
- NEEDS: visual deformation, haptic forces, ...

Some (Virtual) Realities

- Fluids (water, wind,…)
- NEEDS: Motion, turbulence (stochastic?), sounds, haptic forces,...

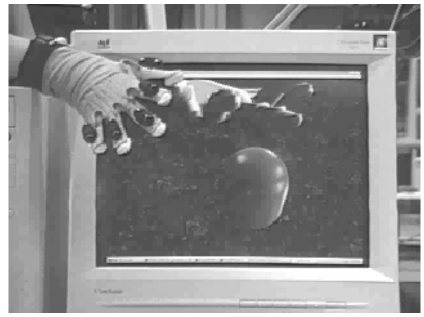
Example 1: Real Wok

Example 1: Virtual Wok



[Doel, Kry and Pai 01]

Example 2: Deformation



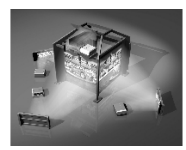
[James + Pai 99]

Input / Output

Visual Displays

- Desktop monitor
- Head mounted display
- Large screen
- Workbench
- Cave
- Fishtank VR

Stanford Responsive Workbench



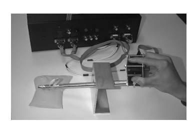
UIC/FakeSpace Cave

Auditory Displays

- Headphones
- Speakers
- Sound synthesis hardware
- Reverberation filters
- Spatialization (HRTF) filters

Haptic Interfaces

PHANToM



Harvard Tactile Display

Logitech IFeel

VTI CyberForce

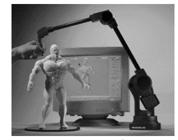
Utah Locomotion Interface

http://www.cs.utah.edu/~jmh/Locomotion.html

Other input devices

- Linkages
- Magnetic trackers
- Gloves
- Body suits
- Tactile sensor pads
- Stereo vision
- Microphones

VTI CyberGlove



MicroScribe

Polhemus

"Real soon now" displays

- Thermal (Ottensmeyer, MIT)
- Smell (e.g., DigiScent)
- ...

Geometric Techniques

1. **Geometry Representation**
2. **Contact Detection**

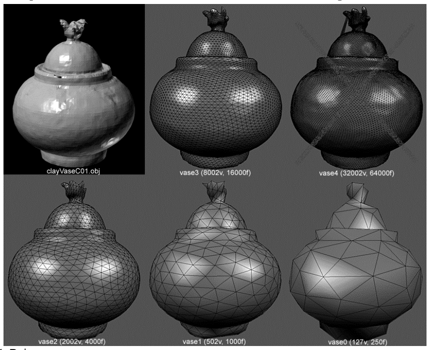
1. Geometry representation

- Polyhedral models
- NURBS and other parametric surfaces
- CSG Models
- Multiresolution and Wavelets
- Subdivision surfaces

Subdivision surfaces

- A way to produce smooth surfaces from a polyhedral model
  - arbitrary topology
  - level of detail
- A natural way to construct multiresolution models (wavelets) over complicated domains [Lounsbery, DeRose, and Warren]

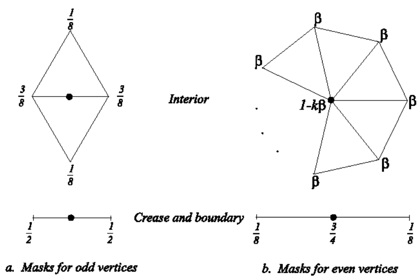
Example: model of real object



Subdivision surfaces

- Many options:
  - Interpolating vs. approximating
  - Triangular vs. quad
- Examples
  - Loop
  - Catmull-Clark
  - Modified Butterfly
  - Displaced subdivision surfaces, Normal meshes, …

Refinement rules for Loop surface



[From: Zorin & Schroder. SIGGRAPH course]

Parametrization

- Can be given piecewise parametrization
- Away from "extraordinary points" parametrization is easy
  - e.g. Catmull-Clark => Bicubic splines
    Loop => quartic box splines
- [Stam 99] shows how to efficiently evaluate Loop and Catmull-Clark surfaces even near extraordinary points

Resources

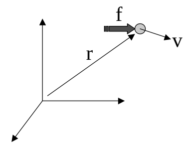
- SIGGRAPH course on subdivision surfaces
  http://www.multires.caltech.edu/pubs/sig00notes.pdf
- Stollnitz, Salesin, DeRose
  "Wavelets for Computer Graphics"
  Morgan Kaufmann

Dynamics and Contact

Overview

- Introduction to dynamics simulation
- Dynamics of a single rigid body
- Contact and Impact
- A framework for Forward Dynamics Algorithms
- Connection to Fast Multibody dynamics and Contact evolution

High school dynamics



- Newton's II Law

$$\dot{r} = v$$

$$\dot{v} = \frac{1}{m} f$$

- -> System of Ordinary Differential Equations (ODEs)

Simulation: Numerical Integration

- Discretize time on a "mesh" of time step h
  - e.g., Forward Euler Method (don't use this!)

$$x(k+1) = x(k) + h\,\dot{x}(k)$$

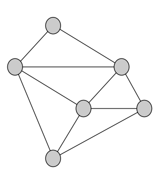
- Issues:
  - order and accuracy
  - step size selection
  - stability

Numerical Integration (continued)

- Achieving high order
  - one-step methods (e.g. RungeKuttaFehlberg45); good for discontinuities in force and motion
  - linear multi-step methods; good for smooth forces
- Achieving stability
  - implicit integrators (e.g., Implicit Runge-Kutta, BDF multistep methods)
- Adaptive step size selection

Rigid body dynamics

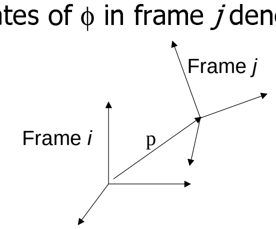
- Particle system view
  - Treat rigid body as a collection of mass points, with distance constraints



- Spatial Vector Dynamics
  - "Eliminate" distance constraints
  - Classical Mechanics, Screw Theory,… with modern notation

Rigid motion and transformation

- Homogeneous Transform $^{i}_{j}\mathbf{E} = \begin{pmatrix} \Theta & \mathbf{p} \\ 0 & 1 \end{pmatrix}$
- Coordinates of ϕ in frame j denoted $^{j}\phi$



Cross product matrix

- Bracket Notation

$$[\omega] = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$$

$$[\omega]\,\mathrm{r} \equiv \omega \times \mathrm{r}$$

Spatial velocity

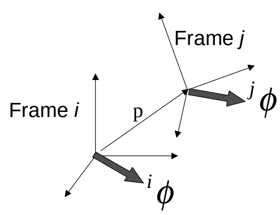
- Spatial velocity ("twist") $\phi = \begin{pmatrix} \omega \\ \mathrm{v} \end{pmatrix}$
  - v linear velocity of frame origin
  - ω angular velocity
- If R is a rigid motion $\dot{R}R^{-1} \equiv \begin{pmatrix} [\omega] & \mathrm{v} \\ 0 & 0 \end{pmatrix}$
- Spatial force ("wrench") $\mathrm{f} = \begin{pmatrix} \mathrm{f_r} \\ \mathrm{f_t} \end{pmatrix}$

Transforming spatial velocities

- Adjoint Transform changes ${}^j\phi$ to ${}^i\phi$

$${}^i\phi = {}^i_j\mathrm{Ad}\,{}^j\phi$$

$${}^i_j\mathrm{Ad} = \begin{pmatrix} \Theta & 0 \\ [\mathrm{p}]\Theta & \Theta \end{pmatrix}$$



Notation...

- Spatial cross product $[\phi] = \begin{pmatrix} [\omega] & 0 \\ [\mathrm{v}] & [\omega] \end{pmatrix}$
- Spatial inertia matrix

$$M = \int \begin{pmatrix} -[r]^2 & -[r] \\ [r] & I \end{pmatrix} dm$$

The notation finally pays off!

- Dynamics of rigid body

$$f = M\dot{\phi} - [\phi]^T M\phi$$

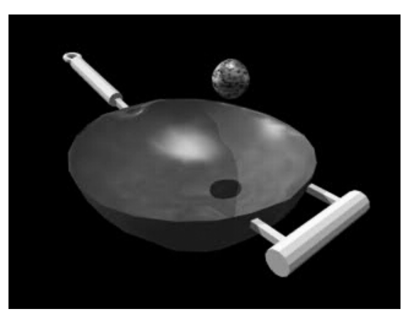
Newton-Euler equations (in body frame)

- Compare with Newton's law for particle

$$f = m\dot{v}$$

- Reference: Pai,Ascher,Kry ICRA 00

Wok Example



Contact interactions

- Two types
  - Impact or collisions:
    relative velocity causes interpenetration
  - Contact (continuous):
    zero relative velocity in normal direction

Rigid body impact

- Real impact is very complex and very fast (~ 50 microseconds)
- We seek approximate models for interactive simulation
- Compatible with rigid body dynamics simulators

Rigid body impact

- *Impulsive models:*
  - Newton, Poisson, Stronge define coefficient of restitution *e,*
    e.g., Newton's restitution $v(t^+) = ev(t^-)$
- *Penalty models:* $m\ddot{x} + f(x, \dot{x}) = 0$
  - Linear harmonic $f(x, \dot{x}) = c\dot{x} + kx$
  - Hunt and Crossley $f(x, \dot{x}) = c x^m \dot{x} + kx$
- Other models: Green's functions [Ullrich&Pai97],...

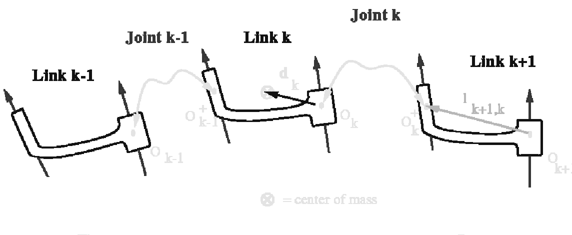
Continuous Contact models

- Linear Complementarity Models
  a = A f + b, "linear"
  a ≥ 0, f ≥ 0, a f = 0 "complementarity"
- For frictionless contact, A is non-negative definite and well behaved.
  => Solutions exist, are unique, and can be efficiently solved by Lemke's method
- For sufficiently large friction, very difficult
- [Lotstedt 84, Baraff, Stewart and Trinkle]

Continuous Contact models

- Penalty models can be used for this as well (an advantage of this method)
  - but can produce stiff equations or penetration
- Deformable models (more accurate)
- Impulsive models can also be used [Hahn88, Mirtich96]
- Contact evolution models
  - treat contact as "generalized joint", and parametrize the degrees of freedom

Multibody Chain



Multibody Dynamics

- Spatial dynamics of link k

$$f_k = {}^{k-1}_{k}\mathrm{Ad}^T \; f_{k-1} + M_k \dot{\phi}_k + b_k$$

- Kinematics of joint k

$$\dot{\phi}_k = {}^{k}_{k+1}\mathrm{Ad} \; \dot{\phi}_{k+1} + H_k \ddot{q}_k + a_k$$

- Forces of joint k

$$\tau_k = H_k^T f_k$$

Extended DAE: $Mx = b$

$$M = \begin{pmatrix} I & M_1 & & & & & & & \\ & I & H_1 & & -\tfrac{1}{2}\mathrm{Ad} & & & & \\ H_1^T & & 0 & & & & & & \\ -\tfrac{1}{2}\mathrm{Ad}^T & & & I & M_2 & & & & \\ & & & & I & H_2 & & \ddots & \\ & & & H_2^T & & 0 & & & \\ & & & \ddots & & & \ddots & \ddots & \\ & & & & & & & \ddots & \ddots \\ & & & & & & \ddots & & \end{pmatrix}$$

[Lubich et al 1992], [Ascher,Pai,Cloutier 1997]

Sparse Gaussian Elimination

$$M = \begin{pmatrix} I & M_1 & & & & & & & \\ & I & H_1 & & -\tfrac{1}{2}\mathrm{Ad} & & & & \\ & & D_1 & & & & & & \\ -\tfrac{1}{2}\mathrm{Ad}^T & & & I & M_2 & & & & \\ & & & & I & H_2 & & \ddots & \\ & & & H_2^T & & 0 & & & \\ & & & \ddots & & & \ddots & \ddots & \\ & & & & & & & \ddots & \ddots \\ & & & & & & \ddots & & \end{pmatrix}$$

Sparse Gaussian Elimination

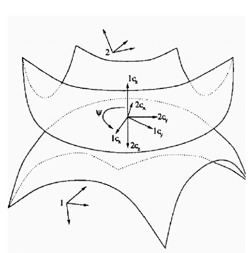
$$M = \begin{pmatrix} I & M_1 & & & & & & & \\ & I & H_1 & & -\tfrac{1}{2}\mathrm{Ad} & & & & \\ & & D_1 & & & & & & \\ & & & I & \hat{M}_2 & & & & \\ & & & & I & H_2 & & \ddots & \\ & & & H_2^T & & 0 & & & \\ & & & \ddots & & & \ddots & \ddots & \\ & & & & & & & \ddots & \ddots \\ & & & & & & \ddots & & \end{pmatrix}$$

ABM [Featherstone 87] = Sparse Gaussian Elimination

$$M = \begin{pmatrix} I & M_1 & & & & & & & \\ & I & H_1 & & -\tfrac{1}{2}\mathrm{Ad} & & & & \\ & & D_1 & & & & & & \\ & & & I & \hat{M}_2 & & & & \\ & & & & I & H_2 & & \ddots & \\ & & & & & D_2 & & & \\ & & & \ddots & & & \ddots & \ddots & \\ & & & & & & & \ddots & \ddots \\ & & & & & & \ddots & & \end{pmatrix}$$

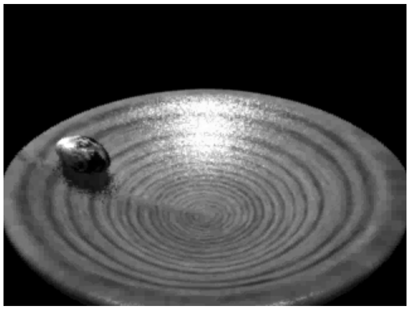
$H_2^T \hat{M}_2 H_2$

Contact Evolution

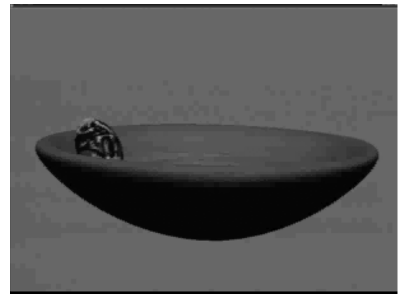


- H_k now more complicated
- But no need for constraint stabilization & distance computation for each integration step
- [Montana 88] contact kinematics for orthogonal nets.
- [Kry,Pai 01] piecewise parametric and subdivision surfaces.

Contact Simulation



Contact Simulation…



Resources

- Baraff and Witkin, SIGGRAPH course notes on Physically-based modeling
  - http://www.cs.cmu.edu/~baraff/
- Murray, Li, Sastry "Mathematical Introduction to Robot Manipulation" CRC Press, 1990
- Featherstone "Robot Dynamics Algorithms" Kluwer 87
- Pai, Ascher and Kry "Forward Dynamics Algorithms for Multibody Chains and Contact" Intl. conf. on Robotics and Automation 2000
  - http://www.cs.ubc.ca/~pai/papers/PaAsKr00.pdf

Sound Simulation

Collaborators:
Kees van den Doel
Derek DiFilippo

Why Sound?

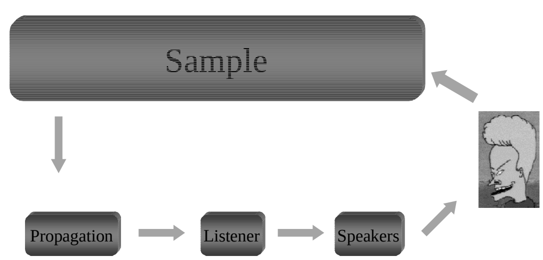
Essential for:

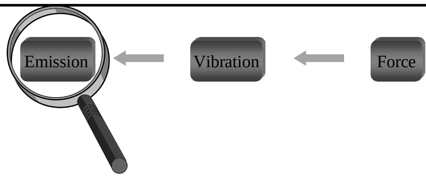
- Scraping
- Rolling
- Sliding
- Rumbling
- Engines

Useful for:

- Striking
- Colliding
- Walking
- Bouncing

Synthesis Method





- Surface-air junction
- Radiation field
- Very complicated

Generalized Sound Cones

- PDE: [redacted]
- Very complicated *A*
- Hard to solve in general
- Good for simple shapes
  - String
  - Bar
  - Plate

Vibration Model

- Assume u obeys wave equation

$$\left(\mathrm{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathrm{u}(\mathrm{x},t) = 0$$

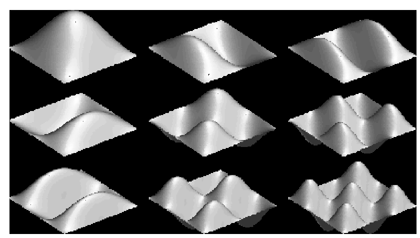
- Solution:

$$\mathrm{u}(\mathrm{x},t) = \sum_{n=1}^{\infty}\left(a_n \sin(\omega_n ct) + b_n \cos(\omega_n ct)\right)\Psi_n(\mathrm{x})$$

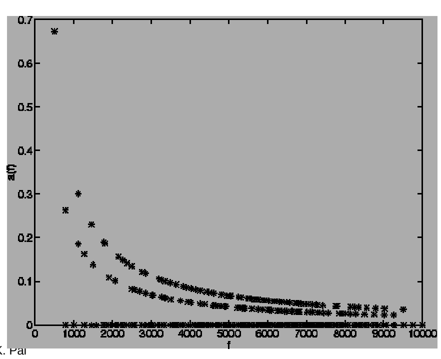
where

$$\left(\mathrm{A} + \omega_n^2\right)\Psi_n(\mathrm{x}) = 0$$

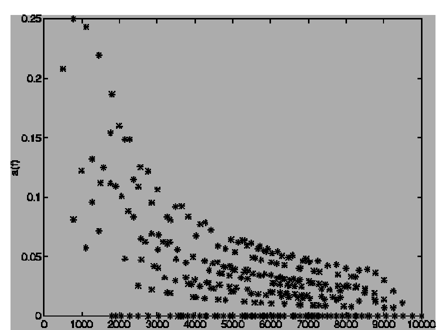
Vibration Modes of Square Membrane



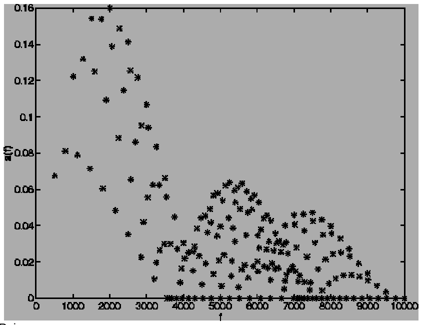
Spectrum of membrane struck at (0.5,0.4)



Spectrum of membrane struck at (0.1,0.4)



Spectrum of membrane struck at (0.1,0.1)



Interactive Sound Synthesis Model

- Impulse response model at boundary vertex

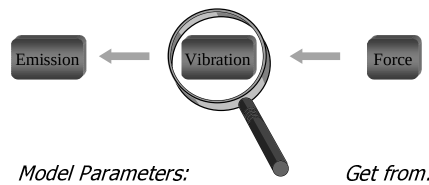
$$p(x,t) = \sum_{i=1}^{M} e^{-b(f_i(x))t} a_i(x) \sin(2\pi f_i(x)t)$$

  $b(f_i)$ is frequency dependent damping, connected to material perception
- Fast synthesis using IIR digital filters (~4 flops per mode)
- Anytime synthesis: render "most important" modes first until time runs out

Sound texture map

- Sound synthesis parameters $f_i(x_j)$, $a_i(x_j)$ and $b_i(x_j)$ are texture mapped onto subdivision surface control vertices
- Interpolate using barycentric coordinates

False color image of audio brightness



Model Parameters:

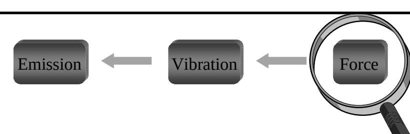
- Frequency spectrum
- Damping spectrum
- Mode shapes

Get from:

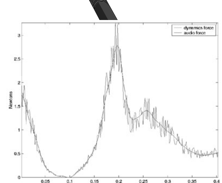
- Compute
- Parameter fitting
- Twiddle

DSP View

- Resonance filters
- Efficient convolution of input forces
- Filter coefficients computable
- Implement as DSP algorithm
- DirectSound demo



- Generate "audio force" from "dynamics force"
  - Sliding
  - Impact
  - Rolling
  - Combustion
  - Abstract



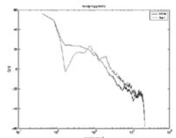
- *Microsimulation*
- *Wavetable*

Micro-simulation of contact force

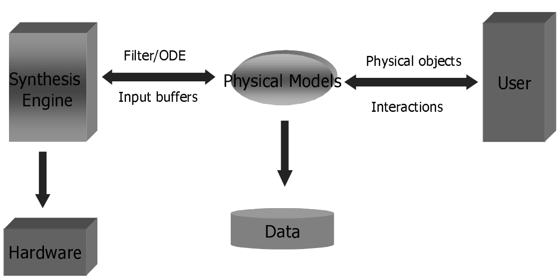
- Impact
  - Impulse-based models can sound too clean
  - Encode "hardness" with duration of pulse
    - e.g., 1- cos(2π/tτ) for 0 < t < τ has correct form
  - Very hard contacts involve multiple bounces
    - use pulse train at dominant resonance frequencies

Micro-simulation of contact force

- Sliding
  - force depends on both roughness and interaction parameters (speed, force)
  - many surfaces well modeled as fractal noise at small scale (spect. independent of speed)
  - capture large scale structure using modal resonance, with frequencies proportional to sliding speed



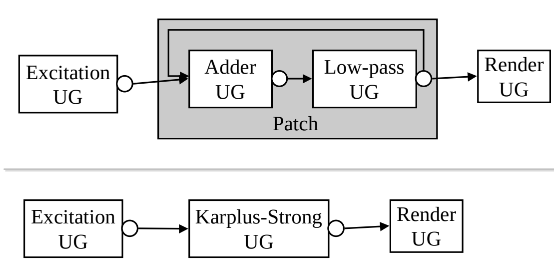
Synthesis API

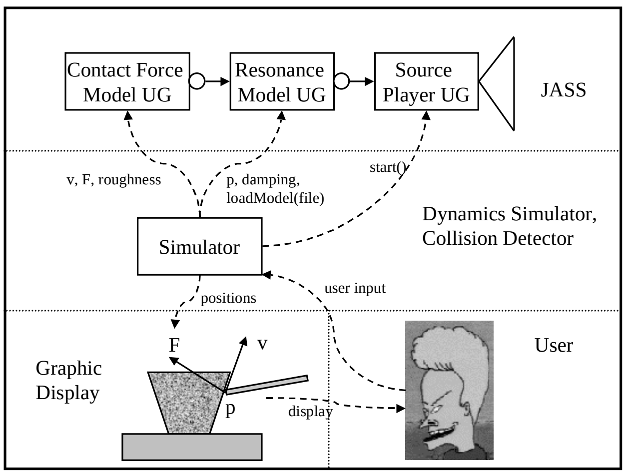


JASS
Java Audio Synthesis System

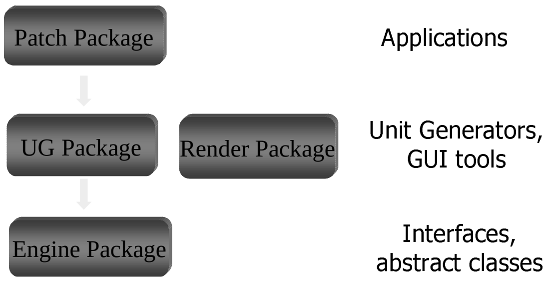
- Pure Java audio SDK
  - For programmers
  - Sound, not music
  - Drive audio with simulation events
  - Interactive (but watch out for OS latencies)
- Unit Generator based
- Free

Unit Generators





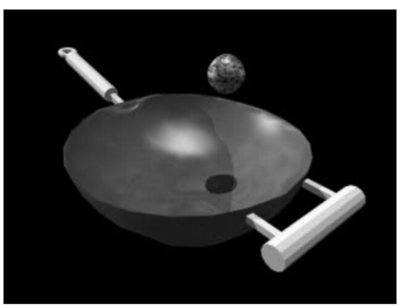
JASS Architecture



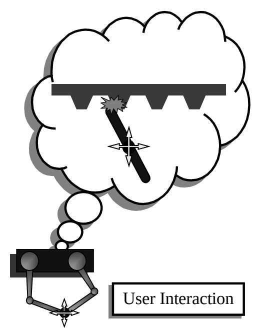
Demos

- Bell
- Bowed String
- Engine
- Scrape
- Bottle

Wok Example

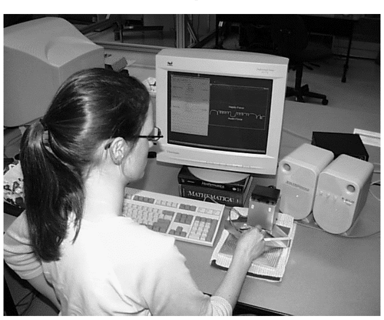


Integrating Audio Haptic Displays



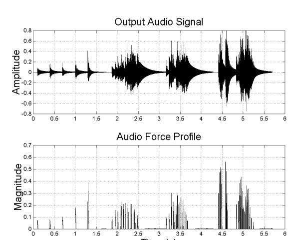
- Contacts are rendered with haptics *and* sound
- Same contact forces drive both haptics and sound
- Haptic force and sound synchronized to < 1ms
- Using
  - Sound synthesis algorithm of [van den Doel & Pai]
  - Pantograph haptic device of [Hayward & Ramstein]
  - Custom DSP controller

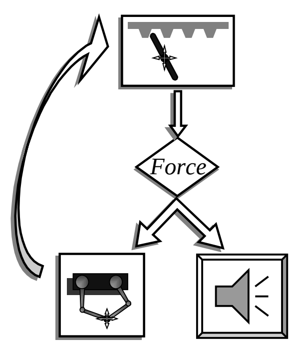
The AHI Audio-Haptic Interface



DiFilippo and Pai UIST 00

Results





A User Study

- Simple pilot to test that 2ms latency lies below perceptual threshold for simultaneity
- Subjects tapped virtual wall, with audio leading or lagging haptics by 2ms
- 2AFC design: choose which came first
- Subjects performed at chance level
- Decay rate appears to be a factor in judging precedence. More studies needed.

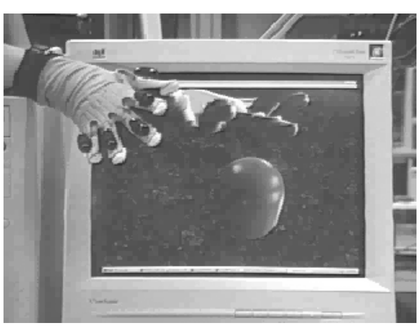
Resources

- Ken Steiglitz *A Digital Signal Processing Primer with applications to Digital Audio and Computer Musi*c. Addison-Wesley, 1996.
- Ken Greenebaum (Ed.) *Audio Anecdotes*, Kluwer 2001 (to appear)
- Free JASS SDK (version 0.9):
  - www.cs.ubc.ca/~kvdoel/jass/jass.html
  - Full source; Build/release/create web scripts

Deformation Simulation

(joint work with Doug James)

Contact Deformation



[James + Pai 99]

Deformation simulation

- Elasticity
- Linear Elastostatic Models
- Green’s Functions
- Capacitance Matrix Algorithms
- Haptic Interaction Issues

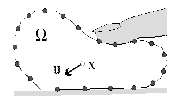
Elasticity

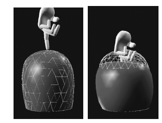
- Dynamics defined at infinitesimal scale
  - force -> stress
  - displacement -> strain
  - Hooke's law relates stress to strain
  - Newton -> Cauchy
- Hyperbolic partial differential equations
- PDE + boundary conditions
  = Boundary Value Problem (BVP)

Linear Elastostatic Models

- Simple
- Stable (time-free)
- Linearity => can exploit principle of superposition
- Captures many "global" aspects of deformation (e.g., incompressibility)

$$G\sum_{k=1}^{3}\left(\frac{\partial^2 u_i}{\partial x_k^2}+\frac{1}{1-2\nu}\frac{\partial^2 u_k}{\partial x_k \partial x_i}\right)+b_i=0$$

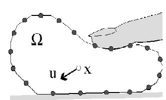




$\nu = 0.5$

Numerical Discretization

- Most problems must be solved numerically
  - Finite Differences
  - Finite Element Method (FEM)
  - Boundary Element Method (BEM)
- FEM => internal discretization; easy to handle anisotropy and inhomogeniety
- BEM=> only boundary discretization; easy to handle mixed boundary conditions



Example: Boundary Elements

N u + b = 0

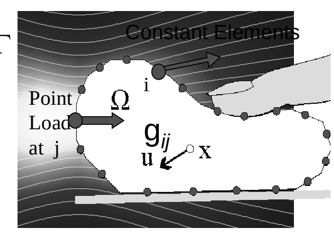
Weaken, Integrate

$$c\,u+\int_\Gamma p^*u\,d\Gamma=\int_\Gamma u^*p\,d\Gamma$$

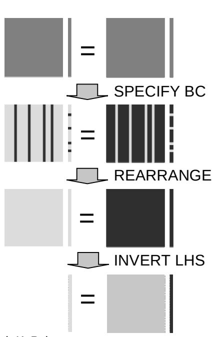
Discretize

H u = G p

$$G\sum_{k=1}^{3}\left(\frac{\partial^2 u_i}{\partial x_k^2}+\frac{1}{1-2\nu}\frac{\partial^2 u_k}{\partial x_k \partial x_i}\right)+b_i=0$$



Green's Functions for Discrete BVP (via BEM)



H u = G p

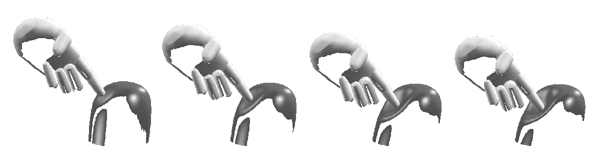
Red	**BV specified**
Yellow	**BV unknown**

$A\,v = -\bar{A}\,\bar{v}$

$v = \underbrace{-A^{-1}\bar{A}}_{\text{Green's Functions}}\bar{v} = \Xi\,\bar{v}$

Fast Solution to BVP with Green's Functions

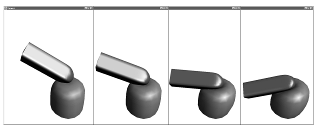
- Usually few specified BV's are nonzero
- If s (out of n) non-zero BVs, O(ns) time to evaluate $v = \Xi\,\bar{v}$



Boundary Condition Type Changes

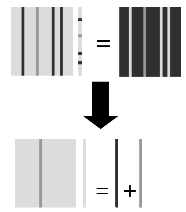
- Problem:
  - if BC changes, have to recompute Ξ
  - Ξ large and dense

- Idea: Exploit coherence



Fast Elastostatic Deformation

- BC change swaps a block column of **A**



$H u = G p$

$A_s v = -\bar{A}_s \bar{v}$

$A_s = A_0 + \delta A_s E^T$

Notation

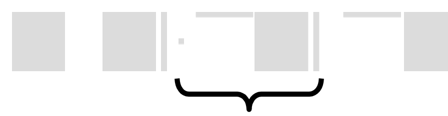
- E is n x s submatrix of Identity
  M E "Extracts" columns from matrix M
  No cost
- Change in matrix $A_s = A_0 + (\bar{A}_0 - A_0)EE^T$
- Changed columns $\delta A_s = (\bar{A}_0 - A_0)E$

$A_s = A_0 + \delta A_s E^T =$ 

Sherman-Morrison-Woodbury

- Idea: Exploit coherence between BVPs
- If $A_s = A_0 + \delta A_s E^T =$ 

$$A_s^{-1} = A_0^{-1} - A_0^{-1}\,\delta A_s \left[I + E^T A_0^{-1}\,\delta A_s\right]^{-1} E^T A_0^{-1}$$



s-by-s capacitance matrix (small)

Capacitance Matrix Algorithm

[James + Pai 99,01]

- Solution to any BVP in terms of Ξ for a Reference BVP

  Using Sherman-Morrison-Woodbury formula:

  $v^{(0)} = [\,\Xi(I-EE^T) - EE^T\,]\,\bar{v}$

  $v = v^{(0)} + (I+\Xi)EC^{-1}E^T v^{(0)}$

  $C = -E^T\Xi E =$ *s*-by-*s* capacitance matrix
- Direct solver w/ fixed solution cost
- Construct, cache and reuse C^{-1}
  - $O(s^3)$ when constraints switch (or better)
  - $O(sn)$ subsequent solves for *s* nonzero BC

- video

Haptic Interaction [James & Pai 01]

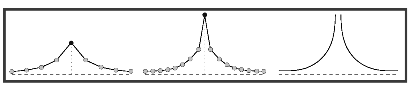
- Need to update contact force at much higher rate (1 KHz)
  - Idea: use faster local model at contact point
  - Related work: [Astley & Hayward 98], [Balanuik 00], [Cavsolglu & Tendick 00]
- Need to support "point contact" abstraction (e.g., for PHANToM)

Capacitance Matrices give Exact Local Models

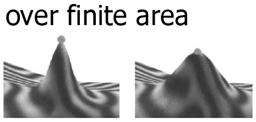
- Consider contacting a free boundary. Forces at only contacted vertices can be computed in $O(s^2)$ time as
  $E^T v = E^T v^{(0)} + E^T (I+\Xi)EC^{-1}E^T v^{(0)}$
- For contact with a single vertex (s=1) simplifies to
  $p_i = E^T v = -C^{-1} u_i = \Xi_{ii}^{-1} u_i$
- So Ξ_{ii} is the effective compliance of vertex

Point Contact abstraction

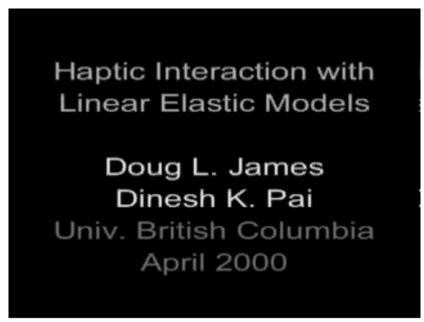
- Real point contact produces infinite tractions (and therefore larger displacements on finer mesh)



- Use vertex masks to distribute displacement over finite area
  [James & Pai 01]



Capacitance Matrix in Haptics



Resources

- Gibson and Mirtich "A Survey of Deformable Models in Computer Graphics" MERL Tech Report TR-97-19, 1997
  - www.merl.com
- James & Pai SIGGRAPH 99 contains a review of elastic models
  - www.cs.ubc.ca/~pai/papers/JamPai99.pdf
- James & Pai "A Unified Treatment of Elastostatic and Rigid Contact Simulation for Real Time Haptics", to appear (approx April 2001) in Haptics-e The Electronic Journal of Haptics Research
  - www.haptics-e.org

Summary

- Geometric techniques
  - models, subdivision surfaces
  - contact detection
- Rigid body dynamics and contact
- Contact sound simulation
- Deformation simulation

Thank you!